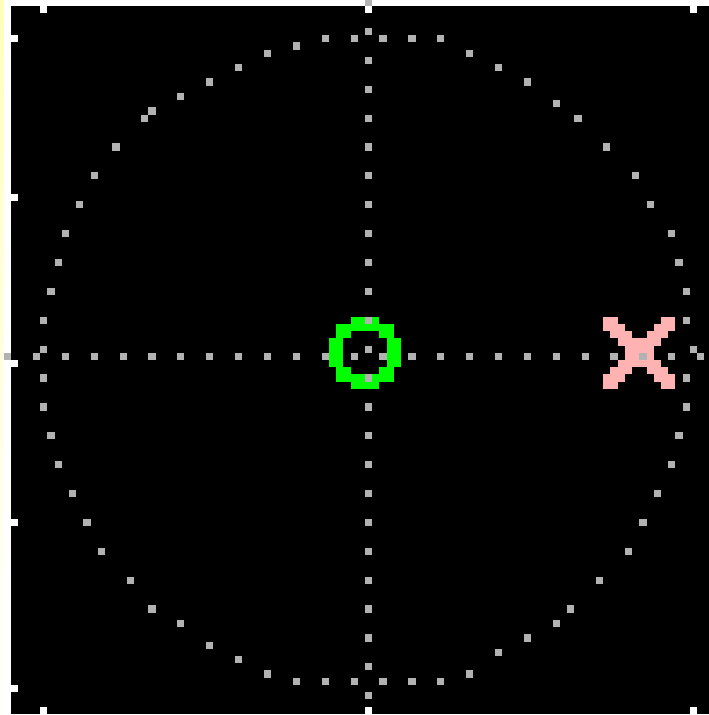


# A7B31ZZS – 6. PŘEDNÁŠKA

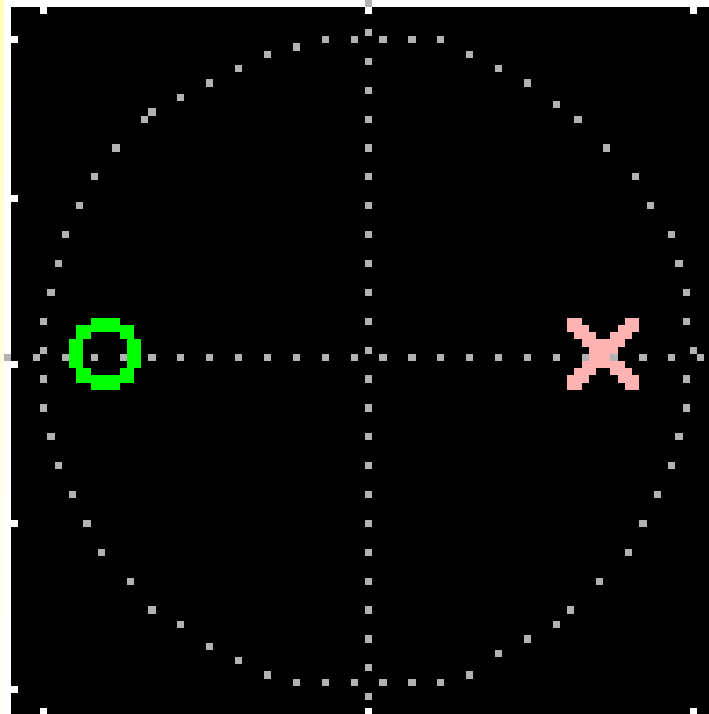
27. října 2014

- **Číslicové IIR filtry vyšších řádu**
  - **filtry se dvěma póly**
  - (filtry s více póly)
  - **řazení filtrů**
- **Aplikace**
  - **banka filtrů (rezonátorů)**
  - **filtrační syntézy s časově prom. filtry**
  - **formantové syntézy (samohlásky, nástroje)**
  - **potlačení šumů**

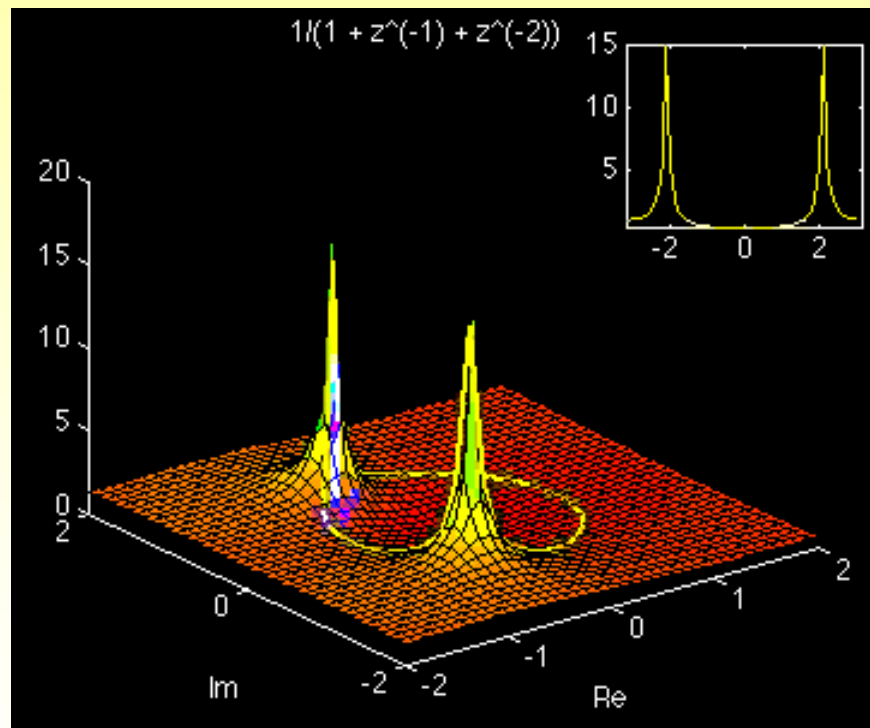
# IIR filtr s jedním pólem (pohyb pólu po reálné ose)



# IIR filtr s jedním pólem a jednou nulou (pohyb nuly a pólu po reálné ose)



# Filtr se dvěma póly - rezonátor



# Filtr se dvěma póly

## pásmová propust IIR

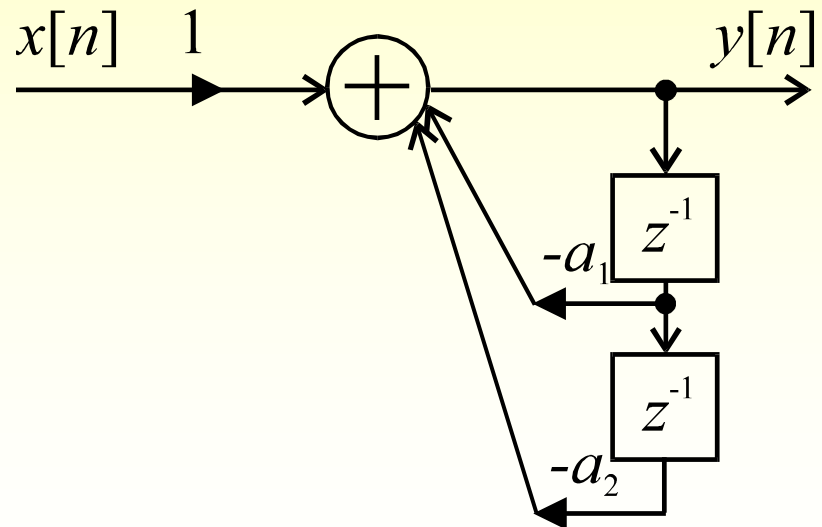
$$y[n] = x[n] - a_1 y[n-1] - a_2 y[n-2]$$

$$H(z) = \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$\Theta_0 = 2\pi \frac{f_0}{f_s}$$

$$a_1 = -2r \cos \Theta_0$$

$$a_2 = r^2$$



# Filtr se dvěma póly

## pásmová propust IIR

$$H(z) = \frac{z^2}{(z - re^{j\theta_0})(z - re^{-j\theta_0})} = \frac{z^2}{z^2 - zre^{-j\theta_0} - zre^{j\theta_0} + r^2}$$

# Filtr se dvěma póly

## pásmová propust IIR

$$\begin{aligned} H(z) &= \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{z^2}{z^2 - zre^{-j\Theta_0} - zre^{j\Theta_0} + r^2} = \\ &= \frac{z^2}{z^2 - zr(\cos \Theta_0 - j \sin \Theta_0 + \cos \Theta_0 + j \sin \Theta_0) + r^2} \end{aligned}$$

# Filtr se dvěma póly

## pásmová propust IIR

$$\begin{aligned} H(z) &= \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{z^2}{z^2 - zre^{-j\Theta_0} - zre^{j\Theta_0} + r^2} = \\ &= \frac{z^2}{z^2 - zr(\cos \Theta_0 - j \sin \Theta_0 + \cos \Theta_0 + j \sin \Theta_0) + r^2} = \\ &= \frac{z^2}{z^2 - 2r \cos \Theta_0 z + r^2} \end{aligned}$$

# Filtr se dvěma póly

## pásmová propust IIR

$$\begin{aligned} H(z) &= \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{z^2}{z^2 - zre^{-j\Theta_0} - zre^{j\Theta_0} + r^2} = \\ &= \frac{z^2}{z^2 - zr(\cos \Theta_0 - j \sin \Theta_0 + \cos \Theta_0 + j \sin \Theta_0) + r^2} = \\ &= \frac{z^2}{z^2 - 2r \cos \Theta_0 z + r^2} = \frac{z^2}{z^2 + a_1 z + a_2} \end{aligned}$$

# Filtr se dvěma póly

## pásmová propust IIR

$$\begin{aligned} H(z) &= \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{z^2}{z^2 - zre^{-j\Theta_0} - zre^{j\Theta_0} + r^2} = \\ &= \frac{z^2}{z^2 - zr(\cos \Theta_0 - j \sin \Theta_0 + \cos \Theta_0 + j \sin \Theta_0) + r^2} = \\ &= \frac{z^2}{z^2 - 2r \cos \Theta_0 z + r^2} = \frac{z^2}{z^2 + a_1 z + a_2} \end{aligned}$$

$$\Theta_0 = 2\pi \frac{f_0}{f_s} \quad a_1 = -2r \cos \Theta_0$$

$$a_2 = r^2$$

# Filtr se dvěma póly

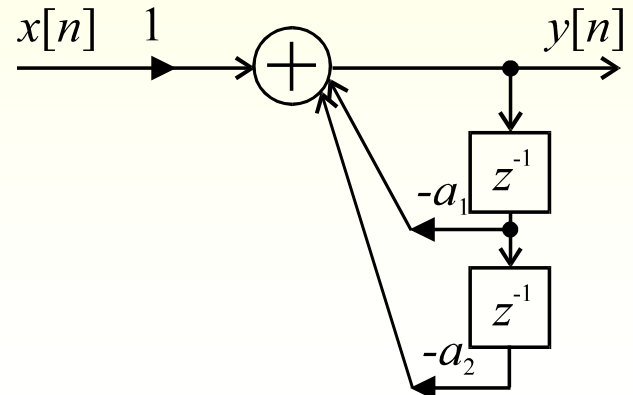
## pásmová propust IIR

$$\begin{aligned}
 H(z) &= \frac{z^2}{(z - re^{j\Theta_0})(z - re^{-j\Theta_0})} = \frac{z^2}{z^2 - zre^{-j\Theta_0} - zre^{j\Theta_0} + r^2} = \\
 &= \frac{z^2}{z^2 - zr(\cos \Theta_0 - j \sin \Theta_0 + \cos \Theta_0 + j \sin \Theta_0) + r^2} = \\
 &= \frac{z^2}{z^2 - 2r \cos \Theta_0 z + r^2} = \frac{z^2}{z^2 + a_1 z + a_2} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}
 \end{aligned}$$

$$\Theta_0 = 2\pi \frac{f_0}{f_s}$$

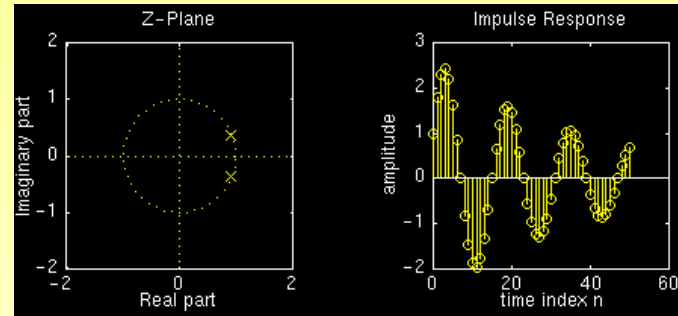
$$a_1 = -2r \cos \Theta_0$$

$$a_2 = r^2$$

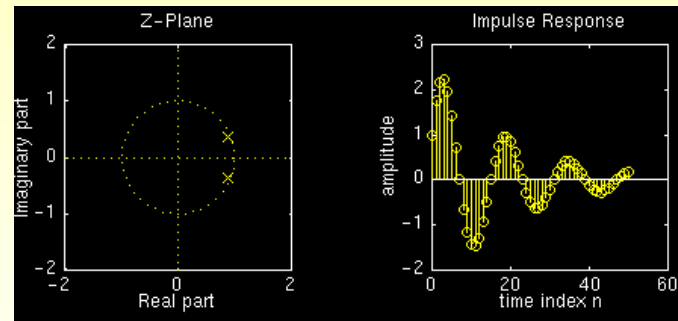


# Filtr se dvěma póly (tlumené oscilace)

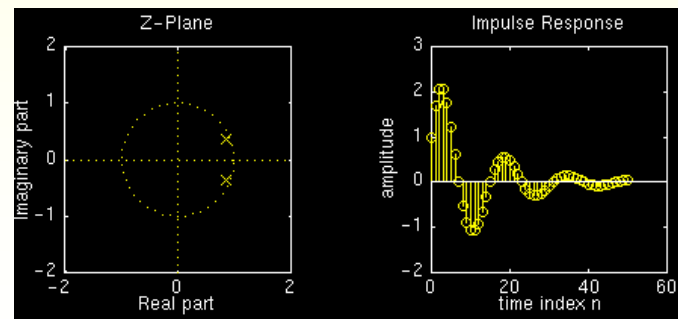
$$\Theta_0 = \frac{\pi}{8}$$
$$r = 0,975$$



$$\Theta_0 = \frac{\pi}{8}$$
$$r = 0,95$$

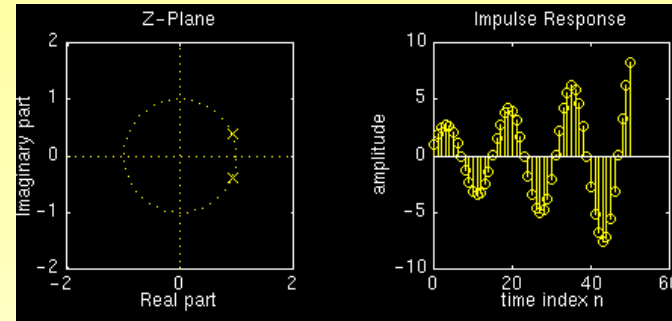


$$\Theta_0 = \frac{\pi}{8}$$
$$r = 0,925$$

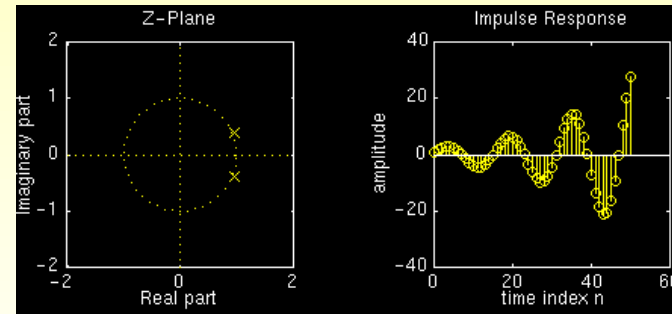


# Filtr se dvěma póly - rezonátor (netlumené oscilace)

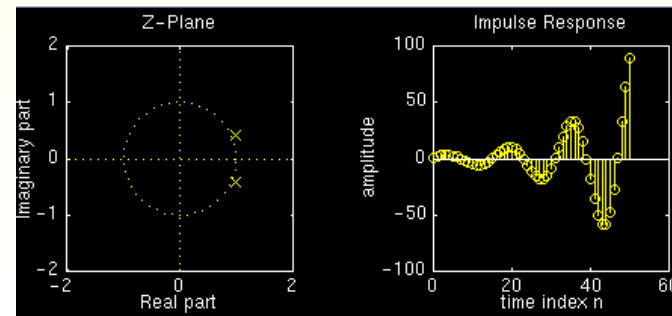
$$\Theta_0 = \frac{\pi}{8}$$
$$r = 1,025$$



$$\Theta_0 = \frac{\pi}{8}$$
$$r = 1,05$$



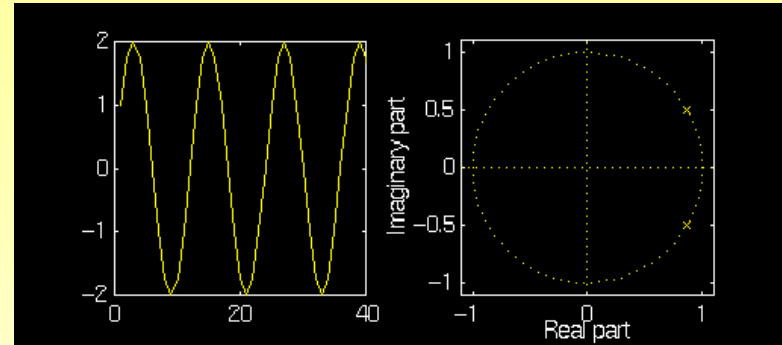
$$\Theta_0 = \frac{\pi}{8}$$
$$r = 1,075$$



# Filtr se dvěma póly - rezonátor (konstantní koeficient $a_2$ )

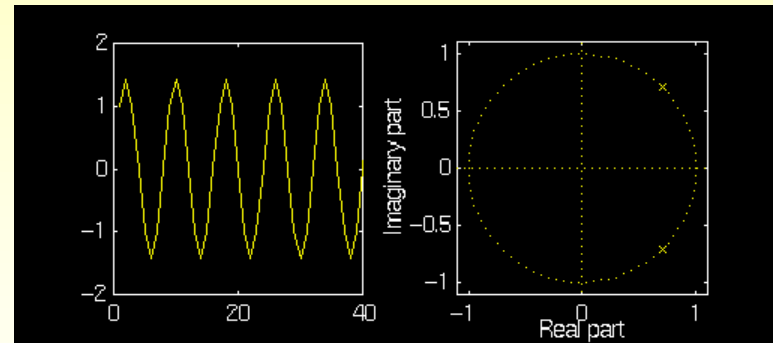
$$a_1 = -2 \cos(\pi / 6)$$

$$a_2 = 1$$



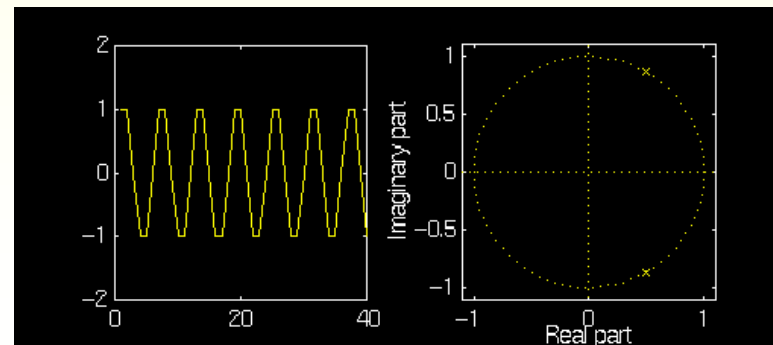
$$a_1 = -2 \cos(\pi / 4)$$

$$a_2 = 1$$



$$a_1 = -2 \cos(\pi / 3)$$

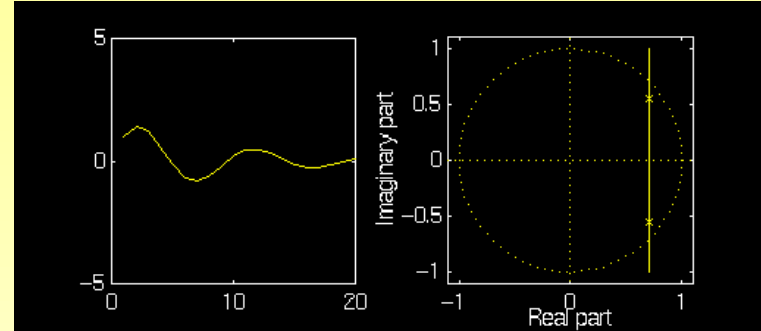
$$a_2 = 1$$



# Filtr se dvěma póly - rezonátor (konstantní koeficient $a_1$ )

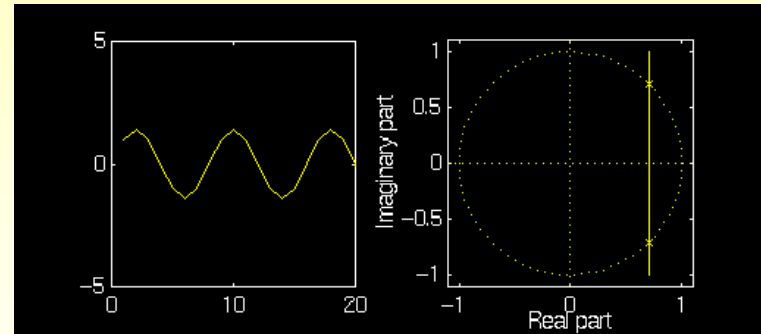
$$a_1 = -2 \cos(\pi / 4)$$

$$a_2 = 0,8$$



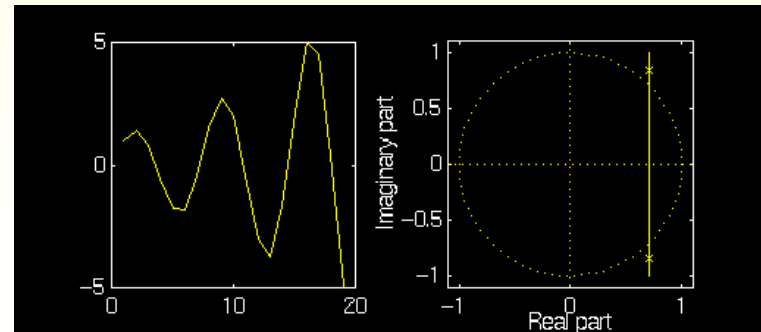
$$a_1 = -2 \cos(\pi / 4)$$

$$a_2 = 1$$



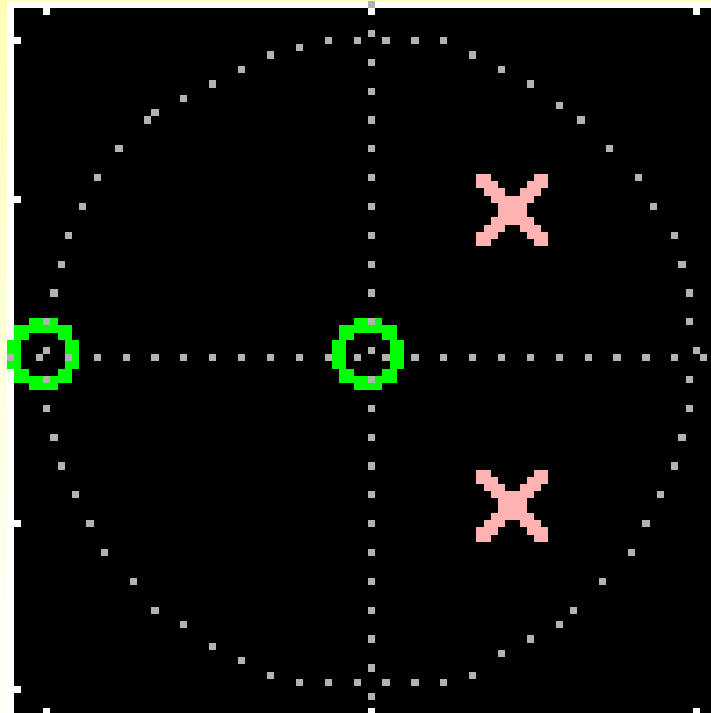
$$a_1 = -2 \cos(\pi / 4)$$

$$a_2 = 1,2$$

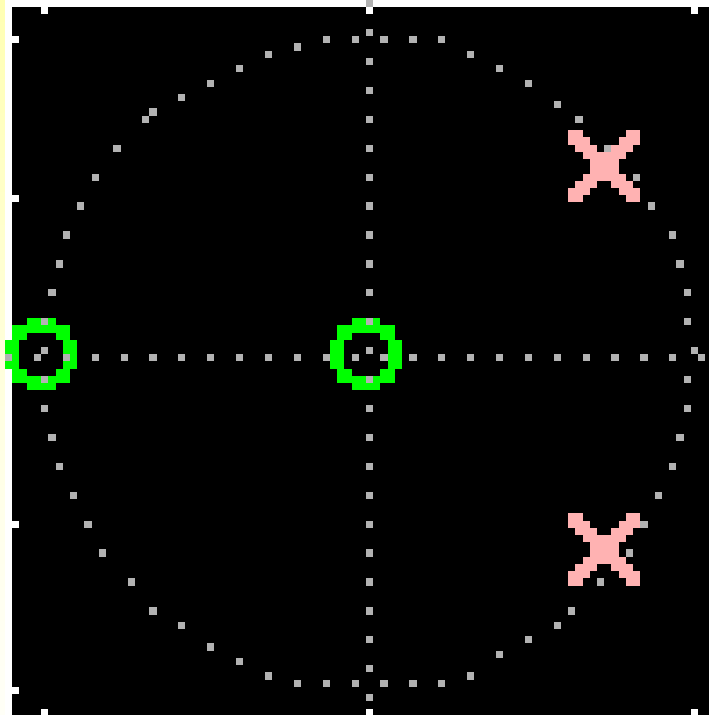


# Pohyb pólů

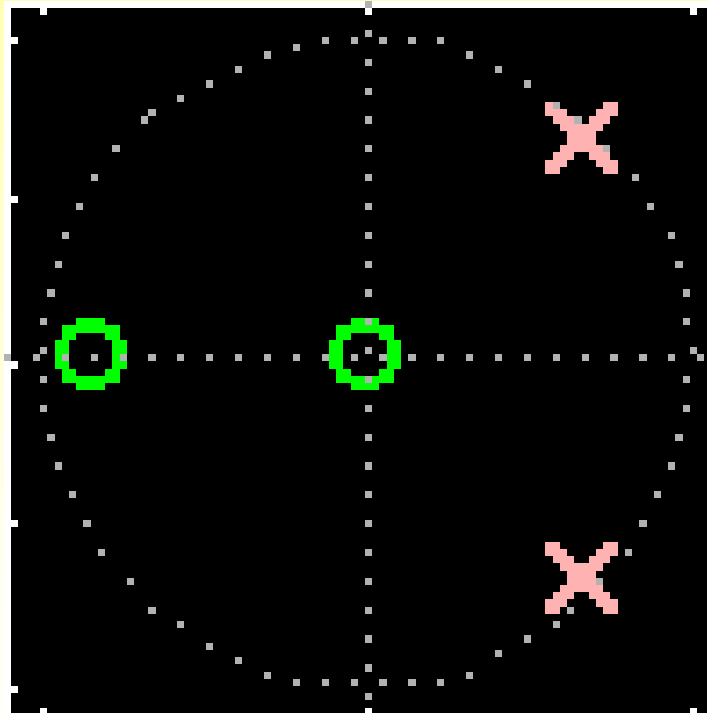
(konstantní úhel; různý poloměr)



# Pohyb pólů po kružnici (různý úhel; konstantní poloměr)

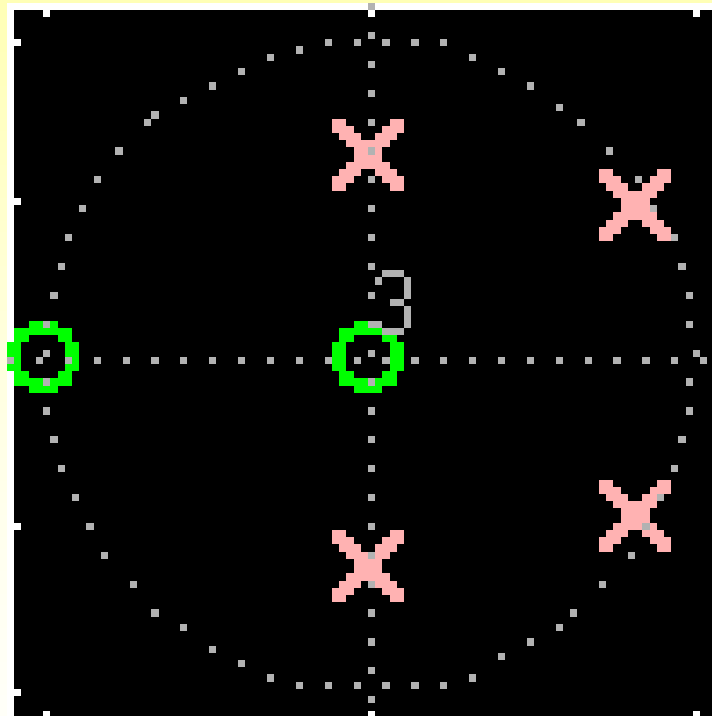


# Pohyb nul IIR filtru (po reálné ose)



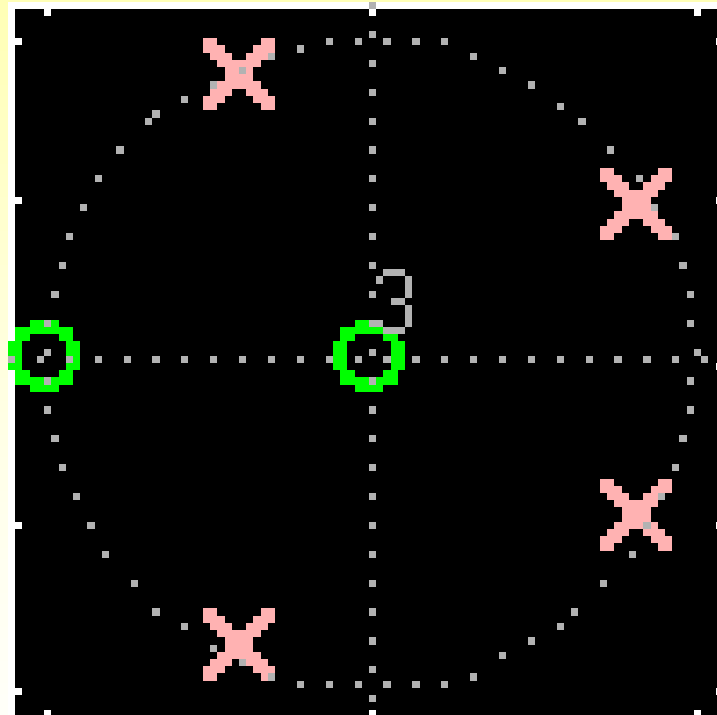
# IIR s více póly

(pohyb vybraných pólů po ose)

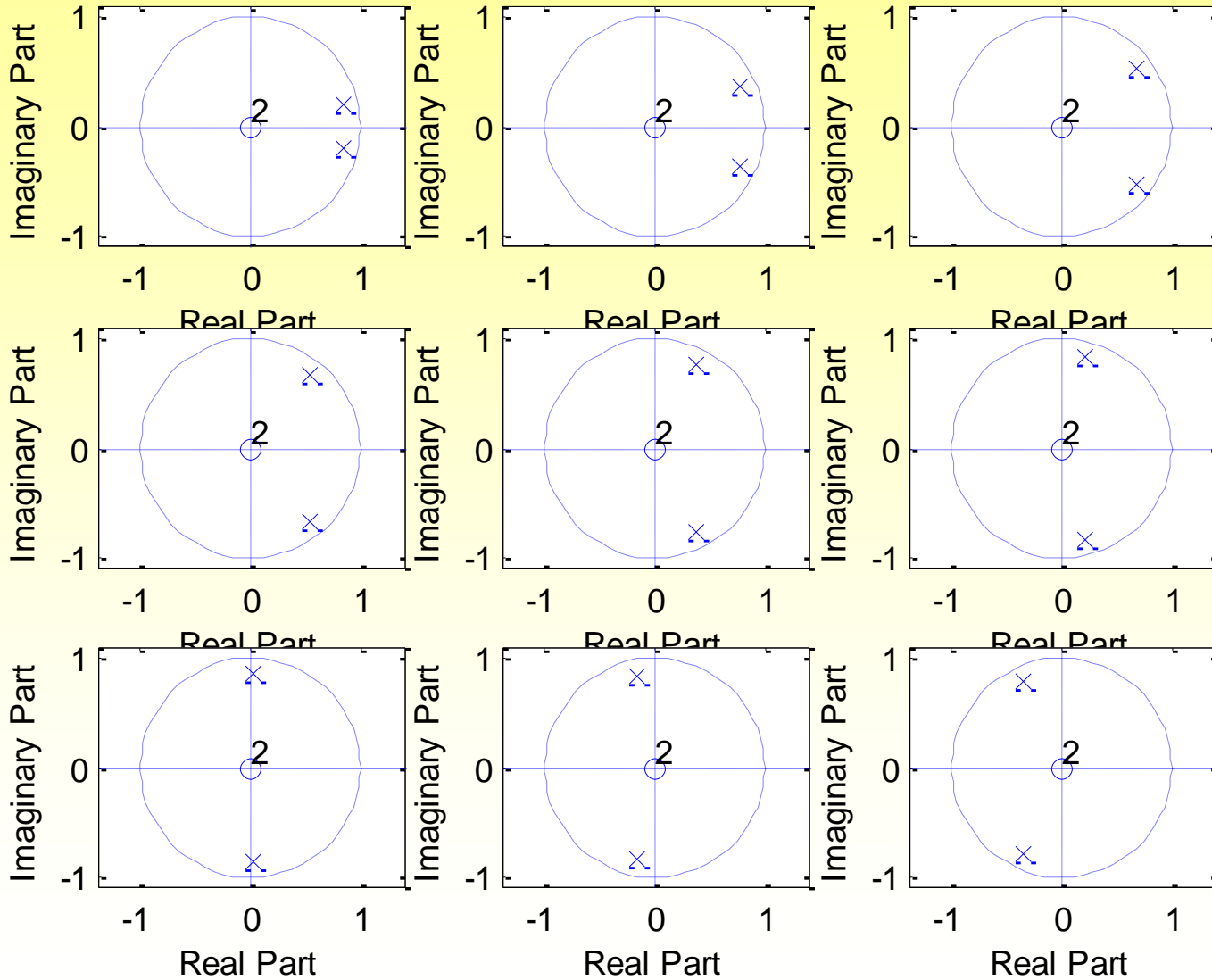


# IIR s více póly

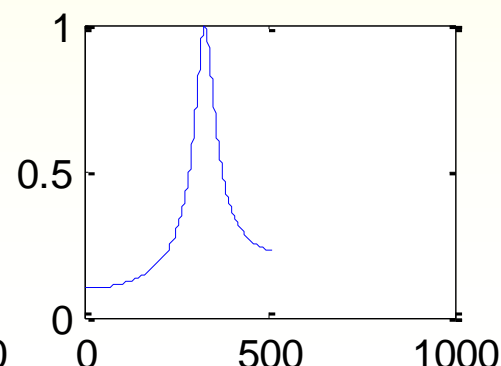
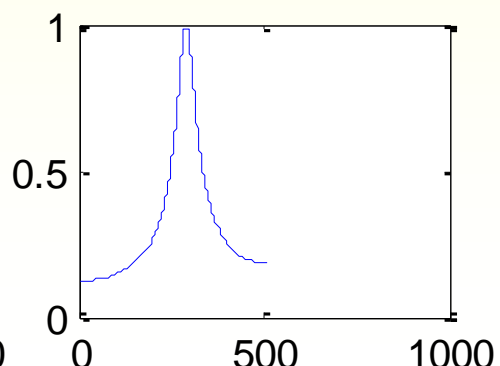
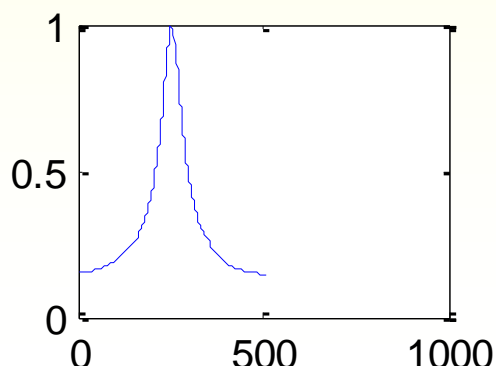
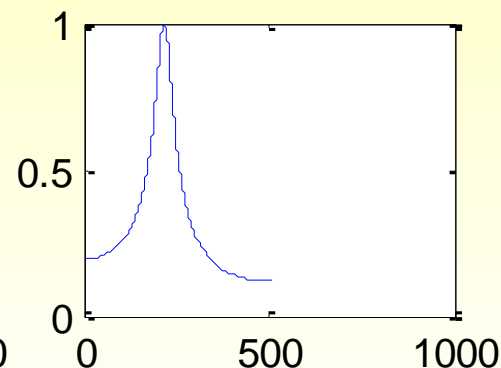
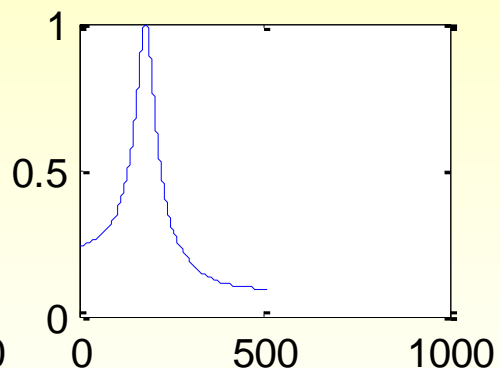
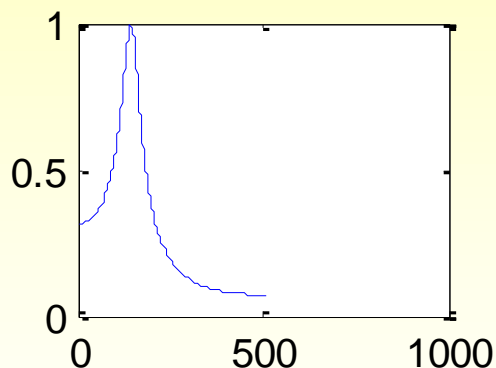
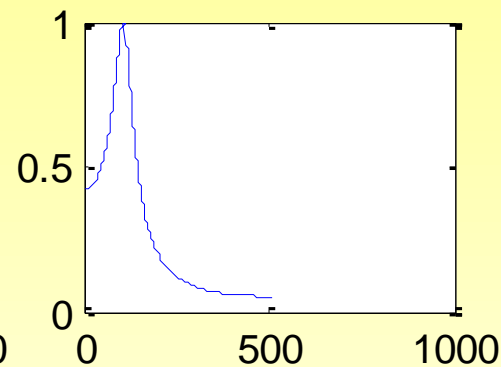
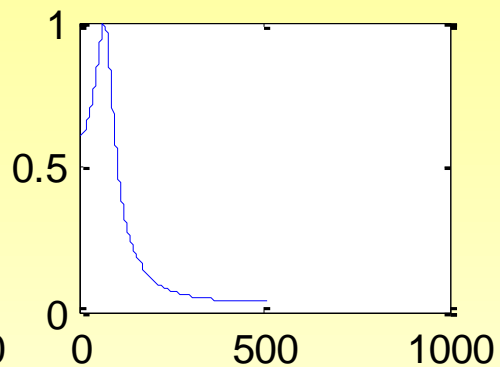
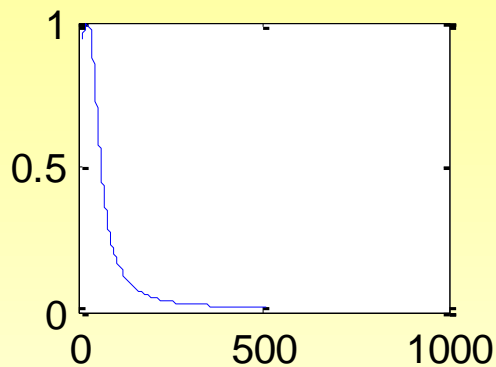
(pohyb vybraných pólů po kružnici)



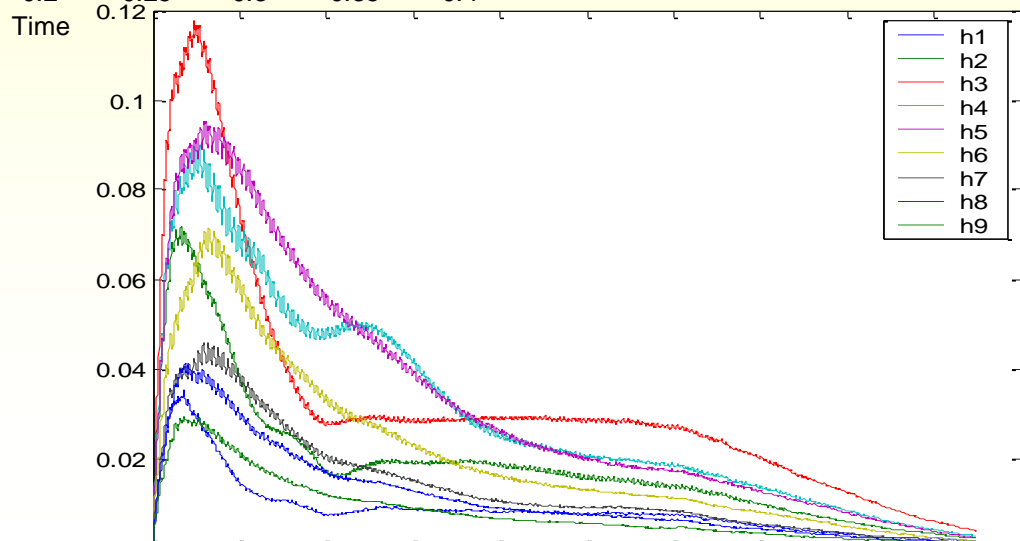
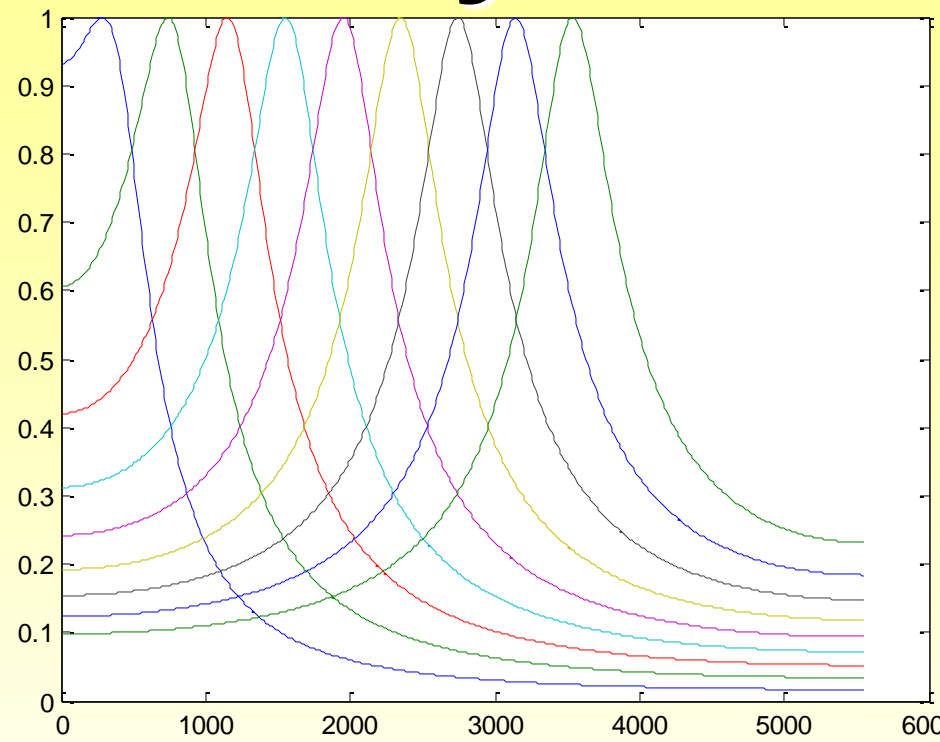
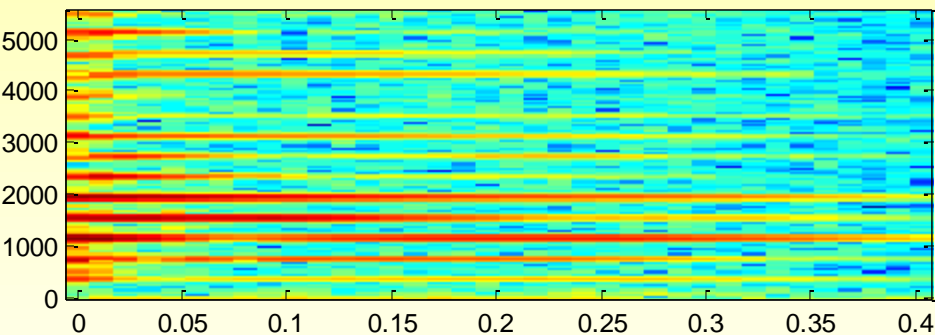
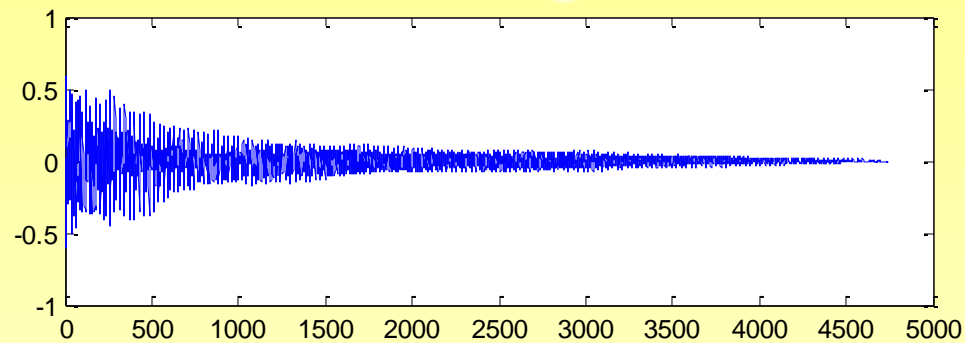
# z-rovina rezonátorů



# Amplitudové charakteristiky rezonátorů

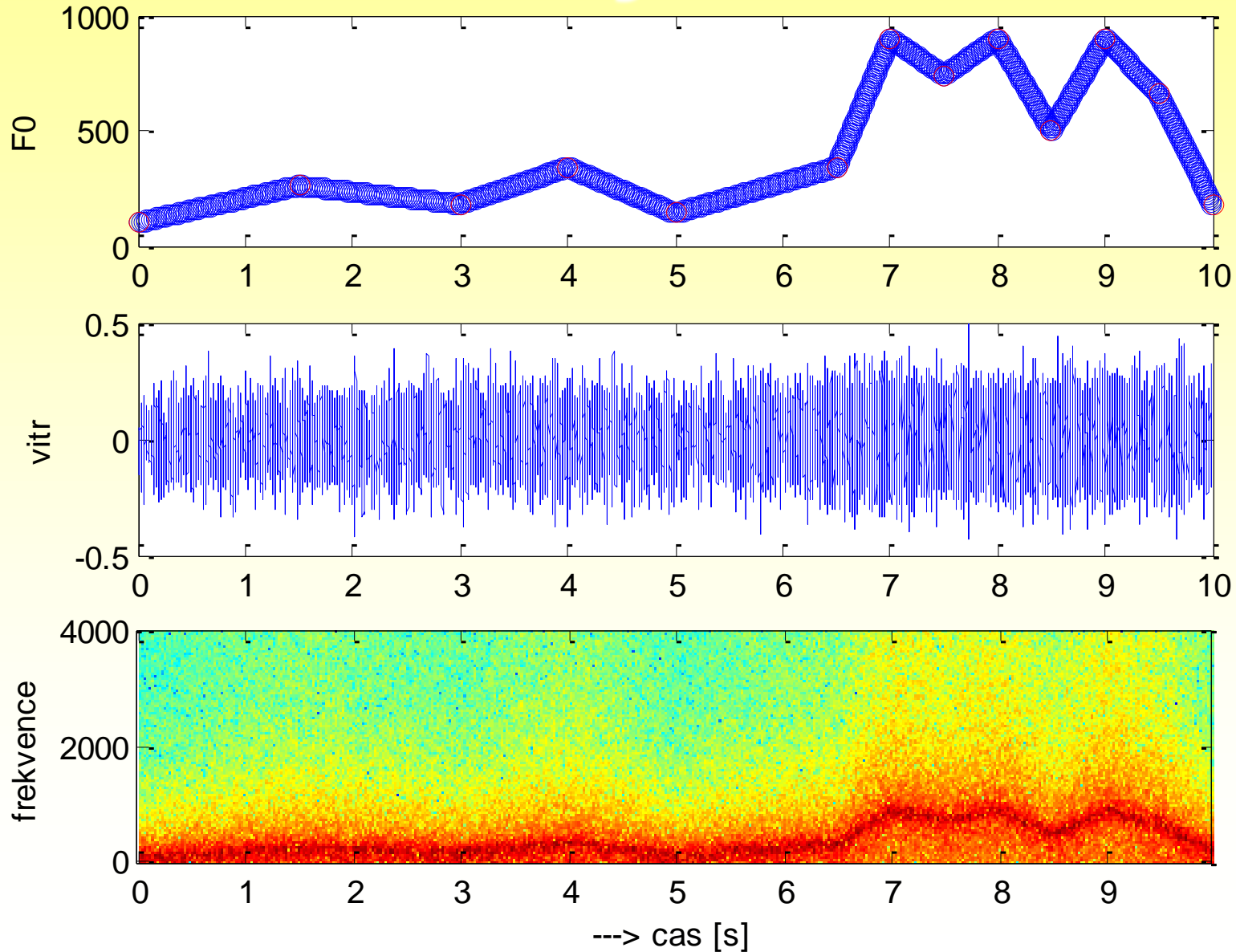


# Analýza harmonických





# Filtrační syntéza větru





# Filtrační syntéza větru

```
clear,  
fs = 8000;          % vzorkovací frekvence [Hz]  
doba = 10;         % doba trvání tonu [s]  
x=2*rand(1,fs*doba)-1; % generování bílého šumu  
nT=0:1/fs:doba-1/fs; % časová osa  
  
% souřadnice řídicích bodů síly větru  
X=[0 .15 .3 .4 .5 .65 .7 .75 .8 .85 .9 .95 1];  
Y=[0 .2 .1 .3 .05 .3 1 .8 1 .5 1 .7 .1];  
Fmin=100; Fmax=900;  
y=(Fmax-Fmin)*Y+Fmin;  
  
% časový průběh interpolované řídicí frekvence  
fr=interp1(X,y,nT/nT(end));
```



# Filtrační syntéza větru

**% navrh rezonatoru**

**B = 100;**

**% sirka pasma rezonatoru**

**R = 1-B\*pi/fs;**

**% vypocet polomer polu**

**a = -2\*R\*cos(2\*pi\*fr/fs);**

**% vypocet koeficientu**

**b0=(1-R)\*sqrt(R\*(R-4\*cos(2\*pi\*fr/fs)+2)+1); %**

**norm.koeficient**

**y=zeros(1,length(x));**

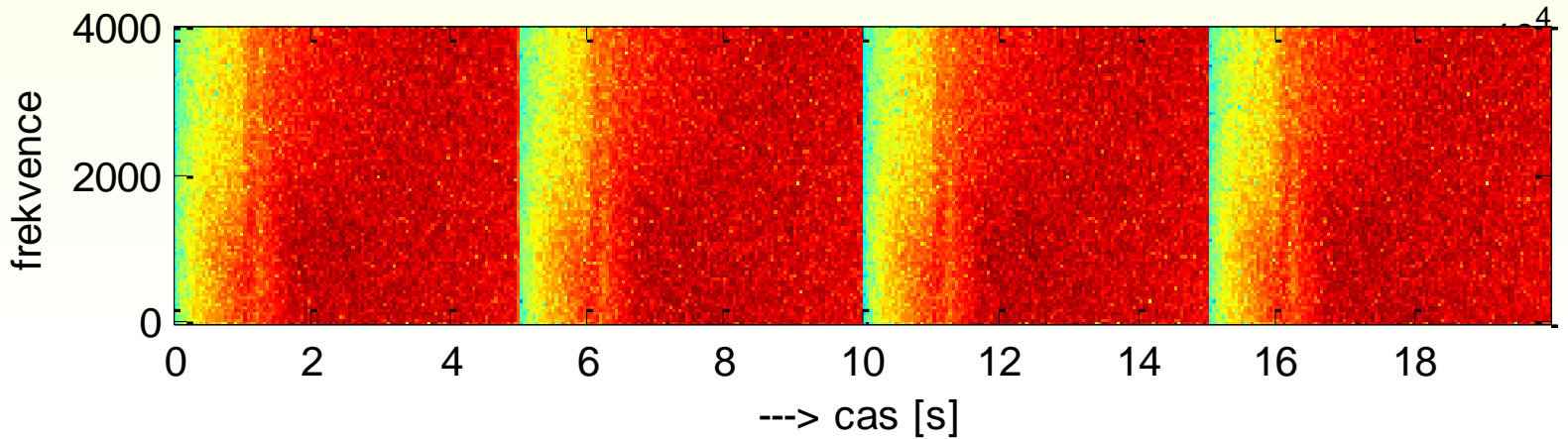
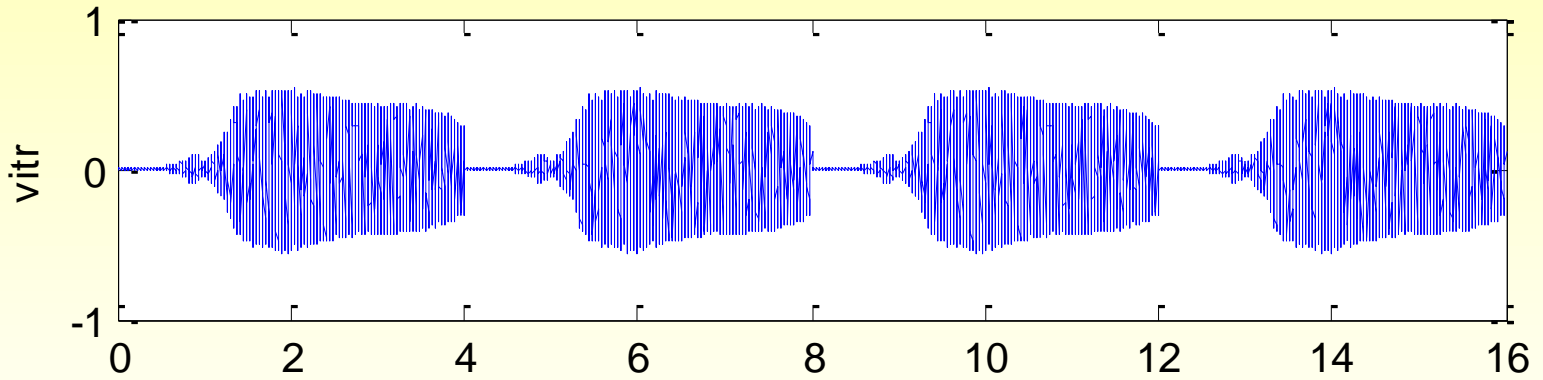
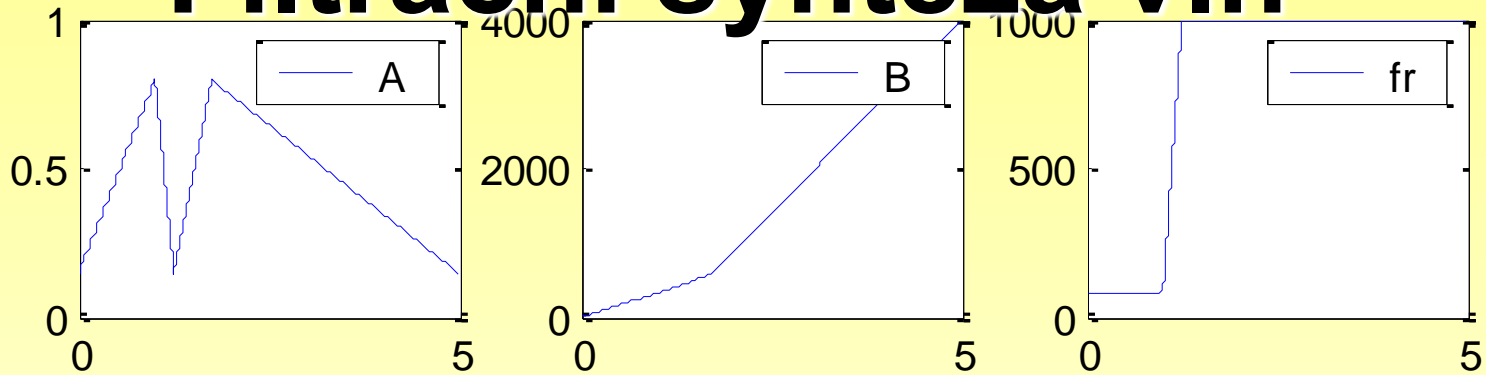
**for n=3:length(x)**

**y(n)=b0(n)\*x(n)-a(n)\*y(n-1)-(R.^2)\*y(n-2);**

**end**



# Filtrační syntéza vln





# Filtrační syntéza vln

**% souradnice ridicich bodu**

**X =[0 .2 .25 .35 1]; % casova osa**

**Y\_A=[0 .8 .15 .8 .1]; % amplituda**

**Y\_B=[0 320 400 600 4000]; % sirka pasma**

**Y\_f=[80 80 1000 1000 1000]; % rezonancni frekvence**

**% casovy prubeh interpolovane ridici amplitudy**

**G=interp1(X,Y\_A,nT/nT(end));**

**% casovy prubeh interpolovane sirky pasma**

**B=interp1(X,Y\_B,nT/nT(end));**

**% casovy prubeh interpolovane ridici rezonancni f**

**fr=interp1(X,Y\_f,nT/nT(end));**



# Filtrační syntéza vln

```
x=2*rand(1,fs*doba)-1;
```

```
for n=3:length(x)
```

```
    R(n) = 1-B(n)*pi/fs;    % vypocet polomer polu
```

```
    a(n) =-2*R(n)*cos(2*pi*fr(n)/fs); % vypocet koeficientu
```

```
    b0(n)=(1-R(n))*sqrt(R(n)*(R(n)-4*cos(2*pi*fr(n)/fs)+2)+1);
```

```
% norm.koeficient
```

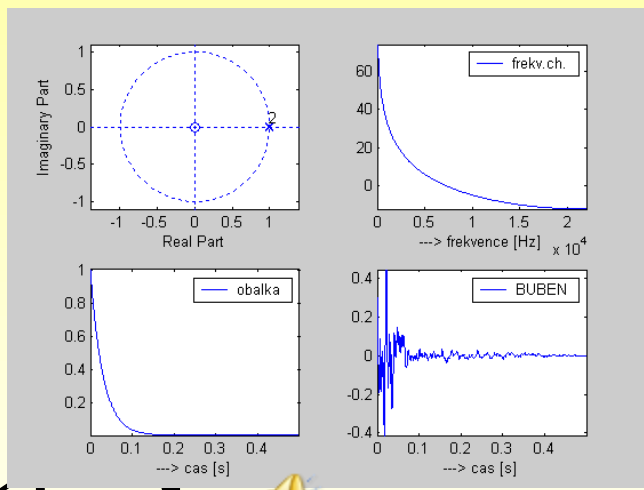
```
    y(n)=G(n)*(b0(n)*x(n)-a(n)*y(n-1)-(R(n).^2)*y(n-2));
```

```
end
```

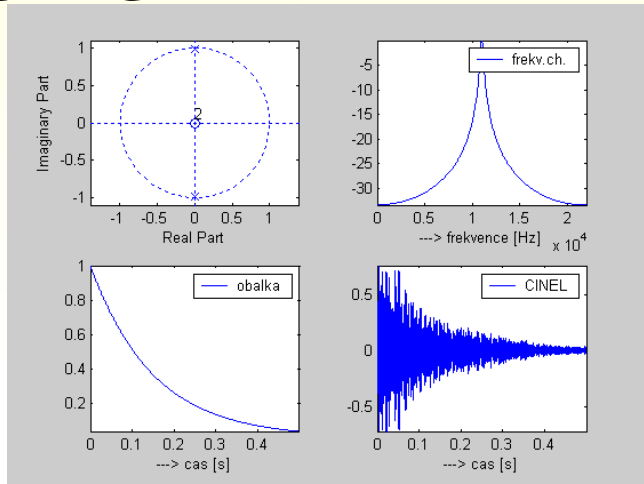
# Filtrační syntéza

Aplikace s šumem rezonátory a obálkami

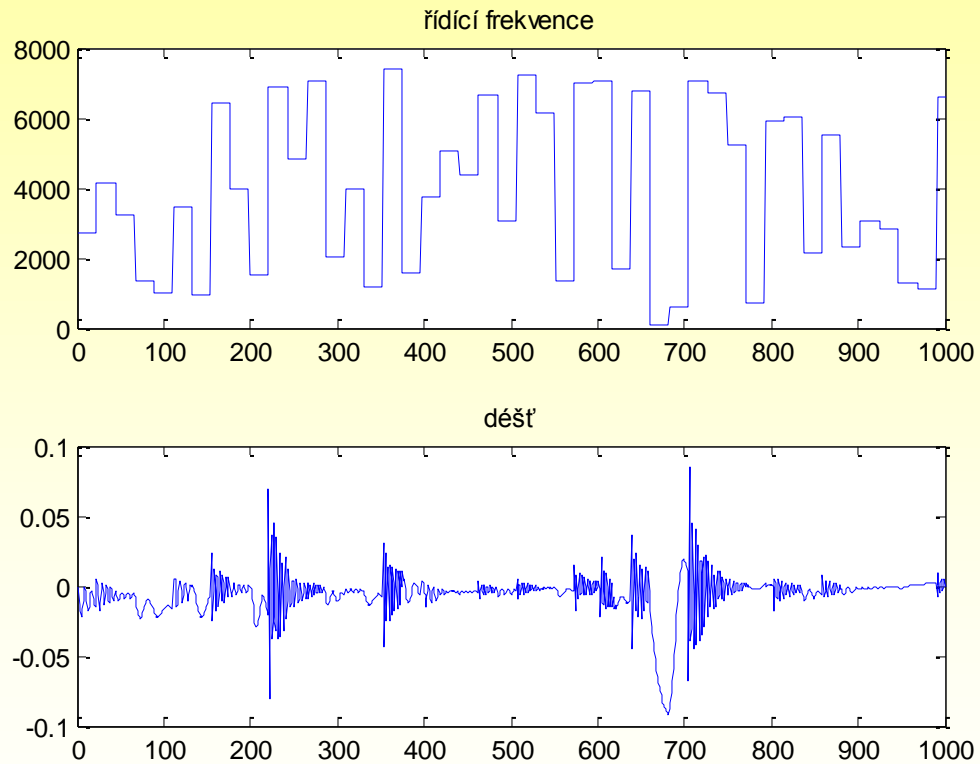
• buben 



• činel 

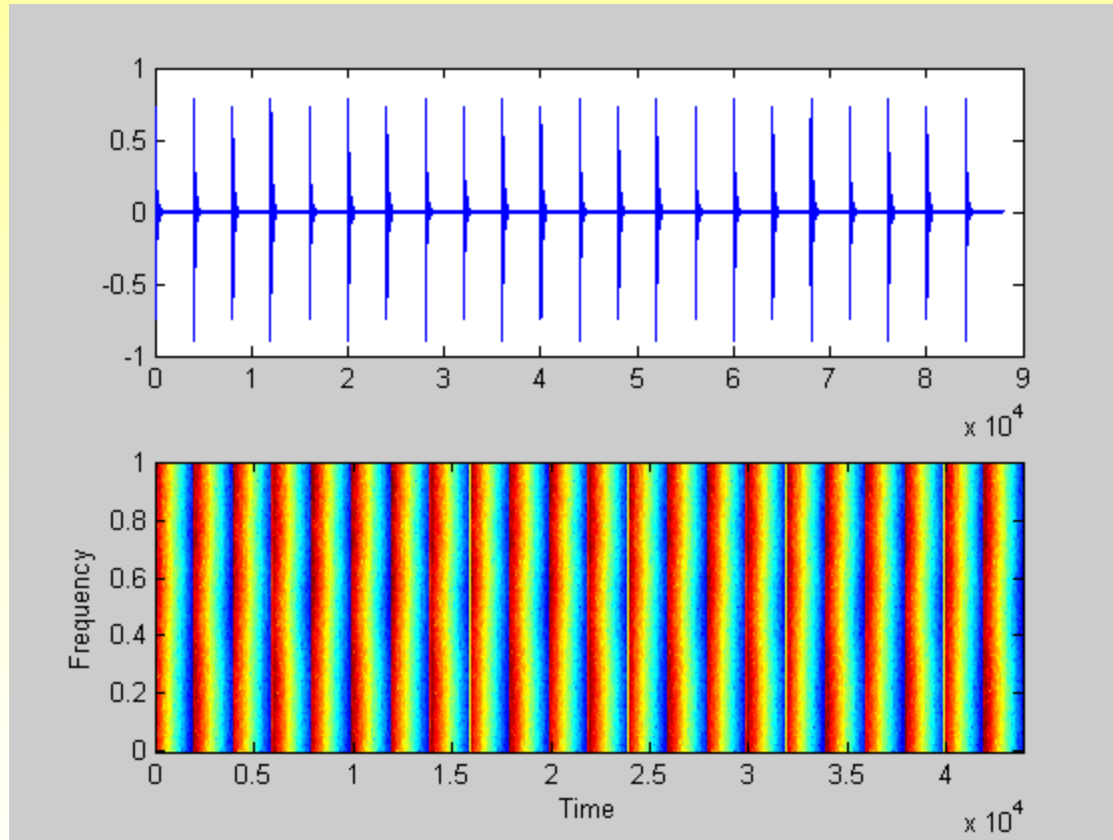


# Filtrační syntéza



# Filtrační syntéza

hodiny

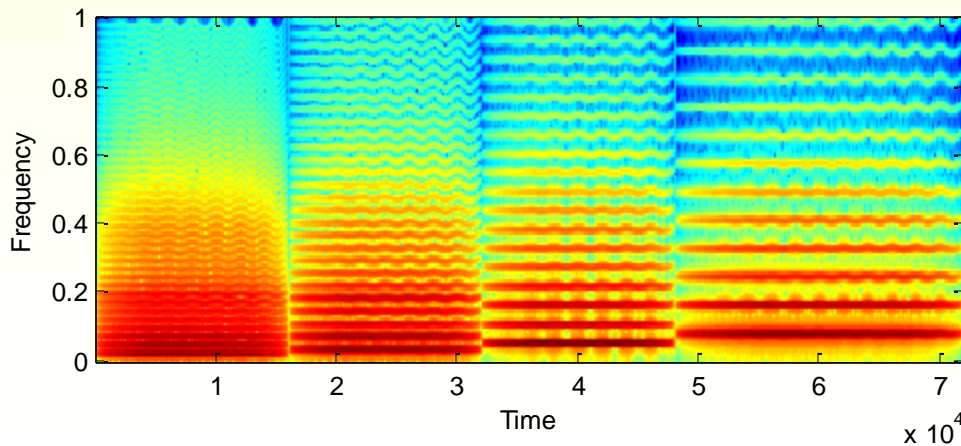
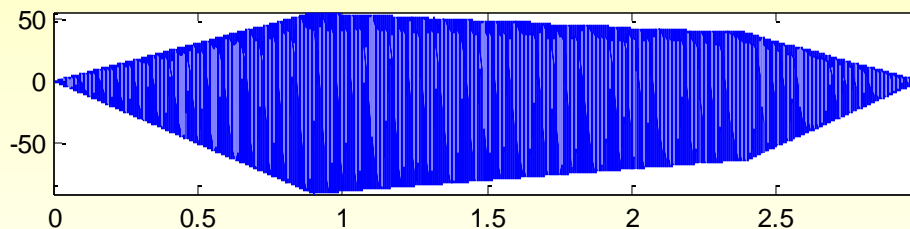
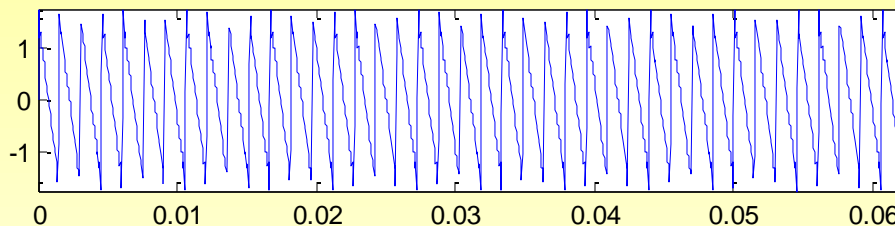
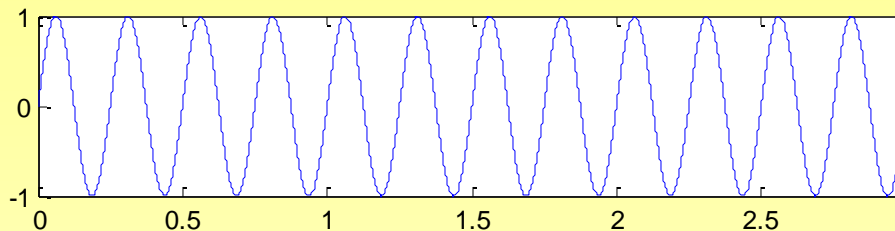


Fr=3500; B = 550;

Fr=3000; B = 750;

# Filtrační syntéza

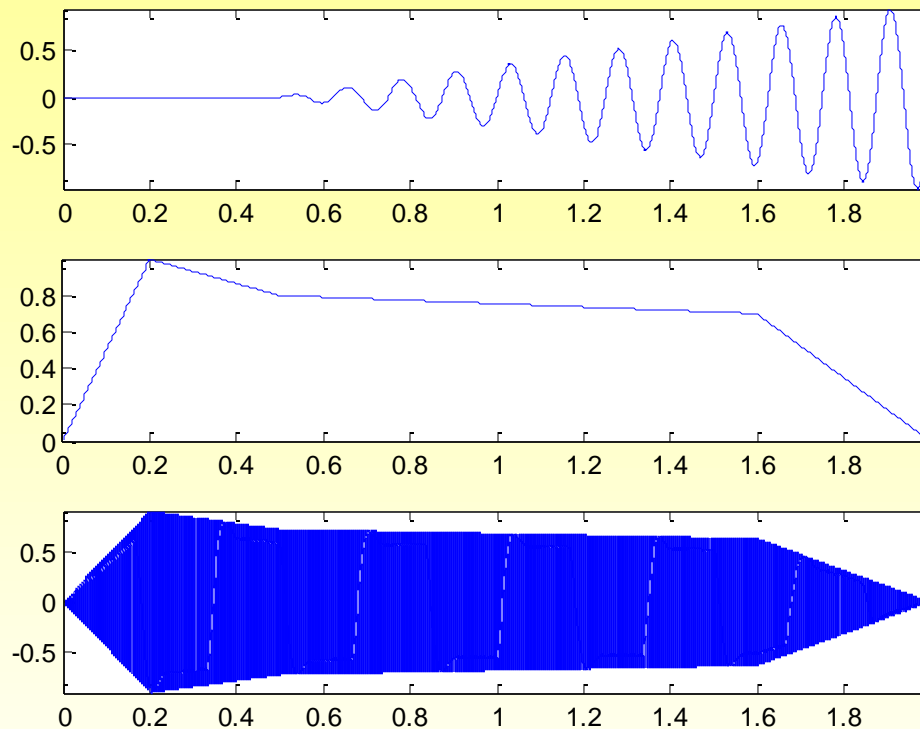
housle



```
% filtr (model housli)
%%%%%%%%%%%%%%
F = [500, 1500, 3000, 4000];
BW = [300, 200, 700, 1500];
```

# Filtrační syntéza

klarinet



% navrh rezonatoru

fr = 900;      % rezonancni frekvence

B = 1200;     % sirka pasma rezonatoru

# Syntéza strojů

vlak



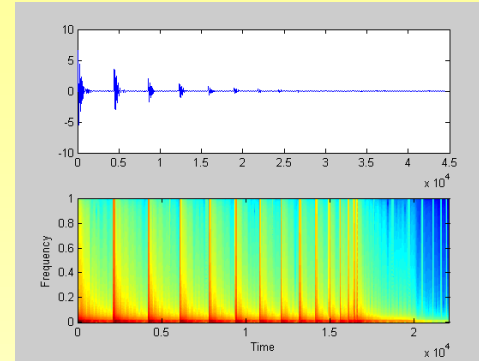
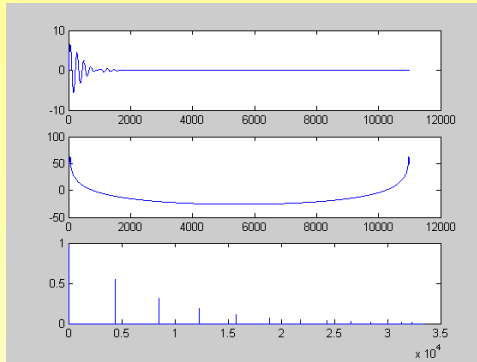
letadlo



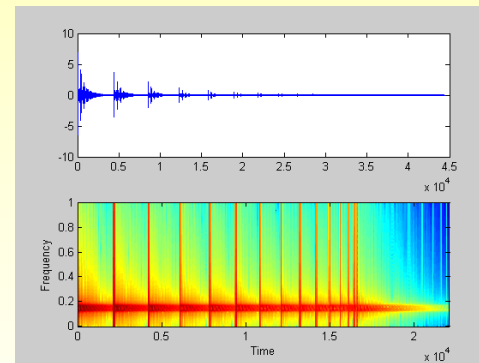
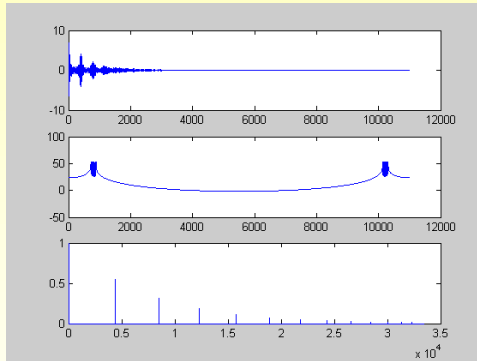
# Syntéza materiálu



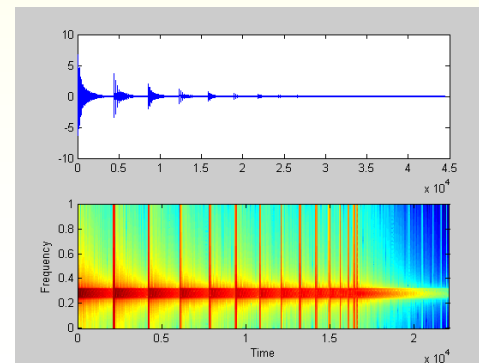
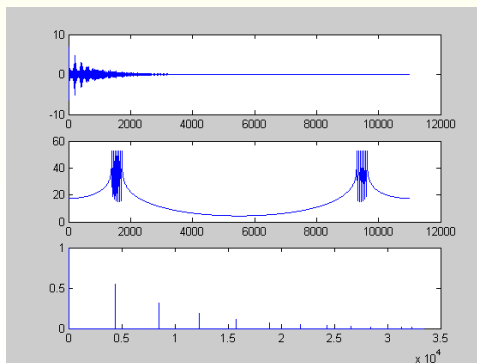
dřevo



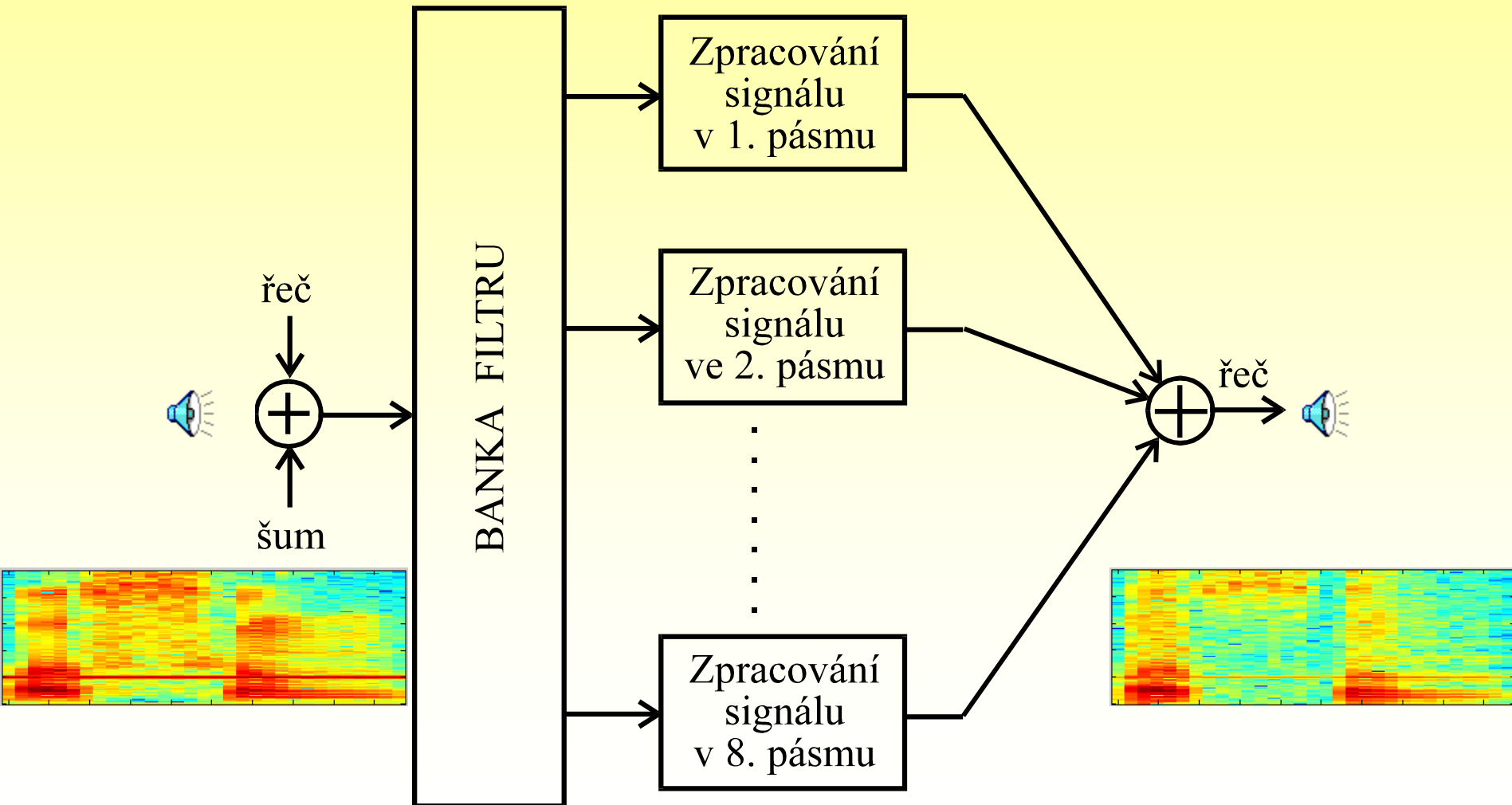
• kov



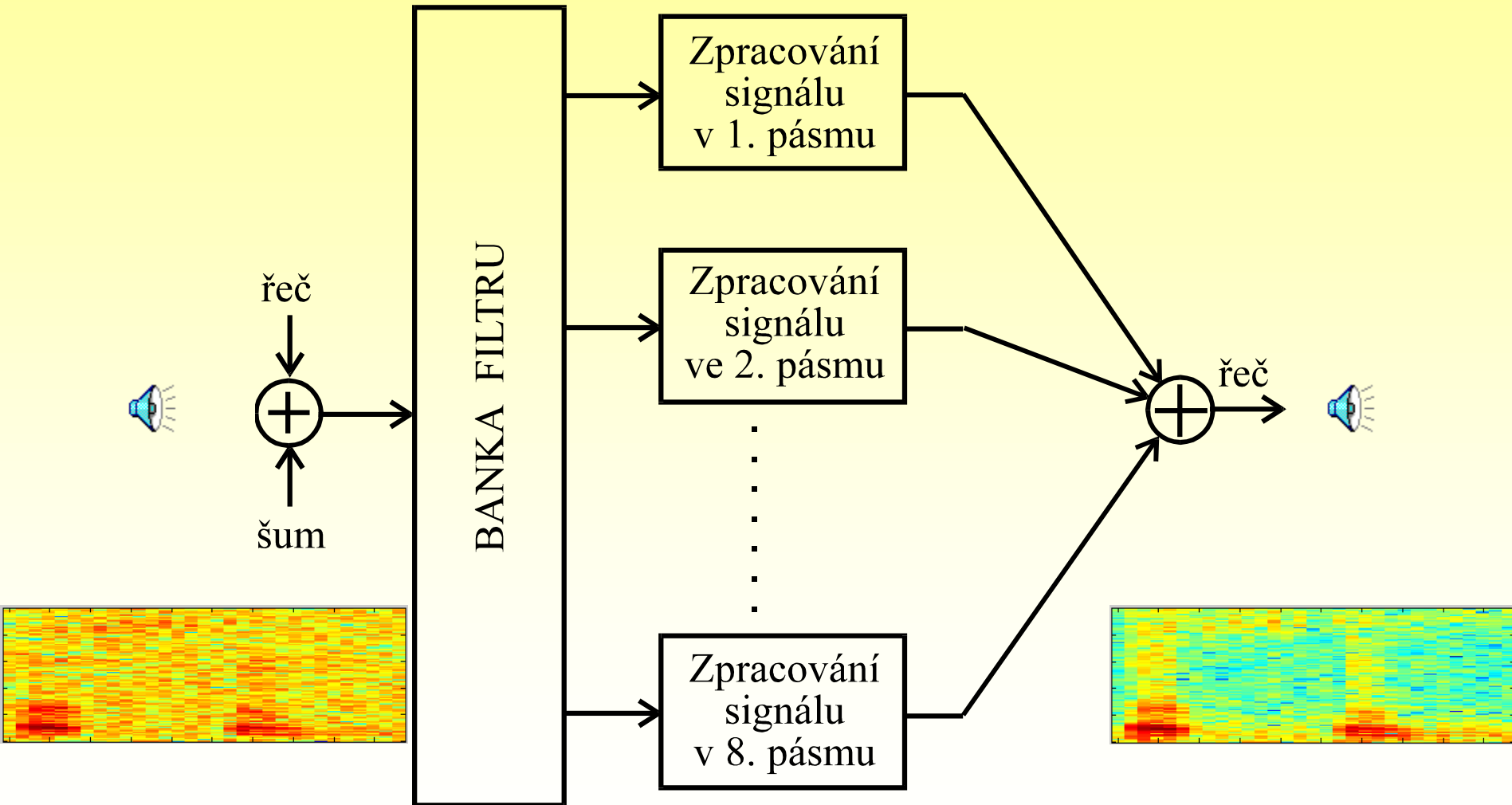
• sklo



# Potlačení šumu v řeči bankou filtrů



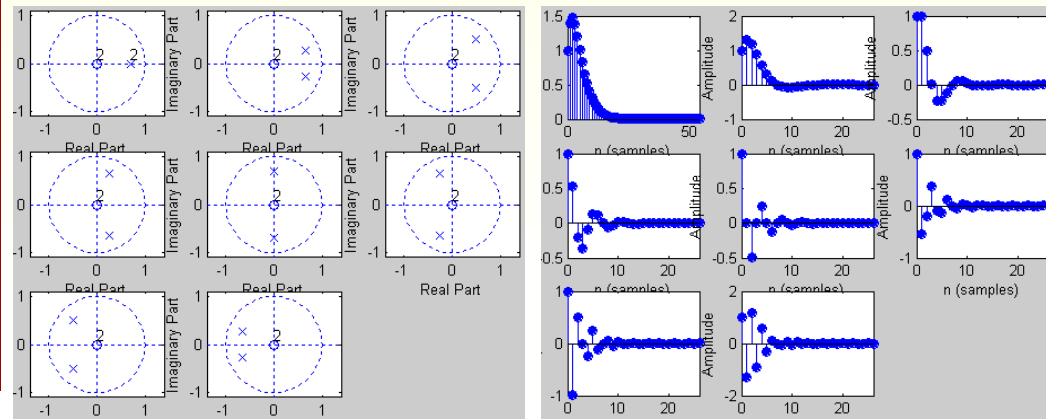
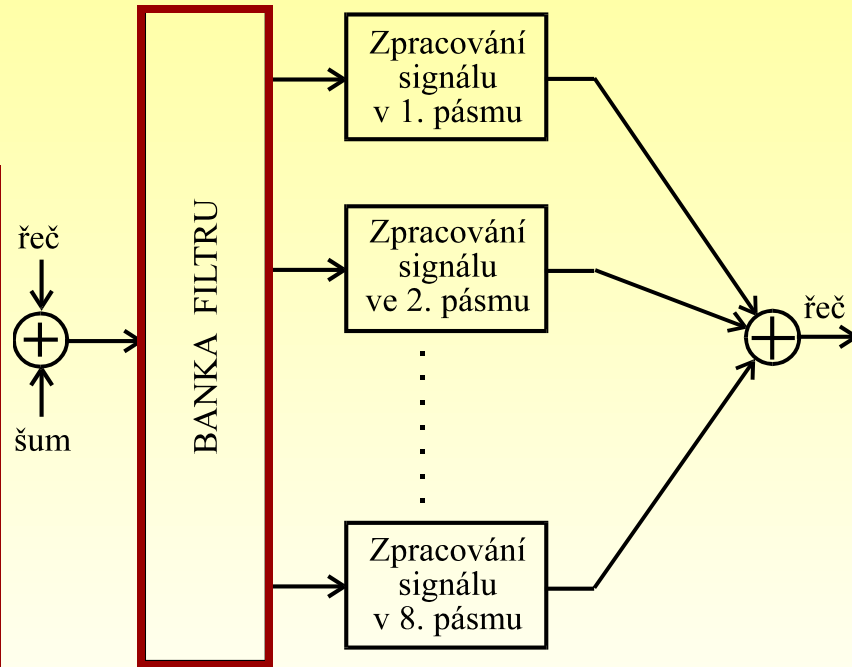
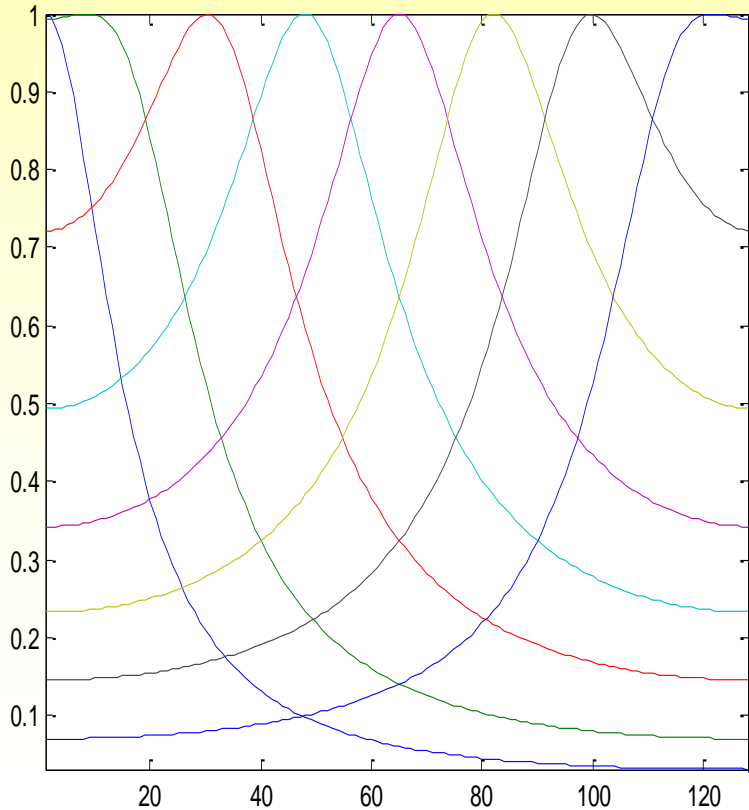
# Potlačení šumu v řeči bankou filtrů



# Potlačení šumu v řeči bankou filtrů

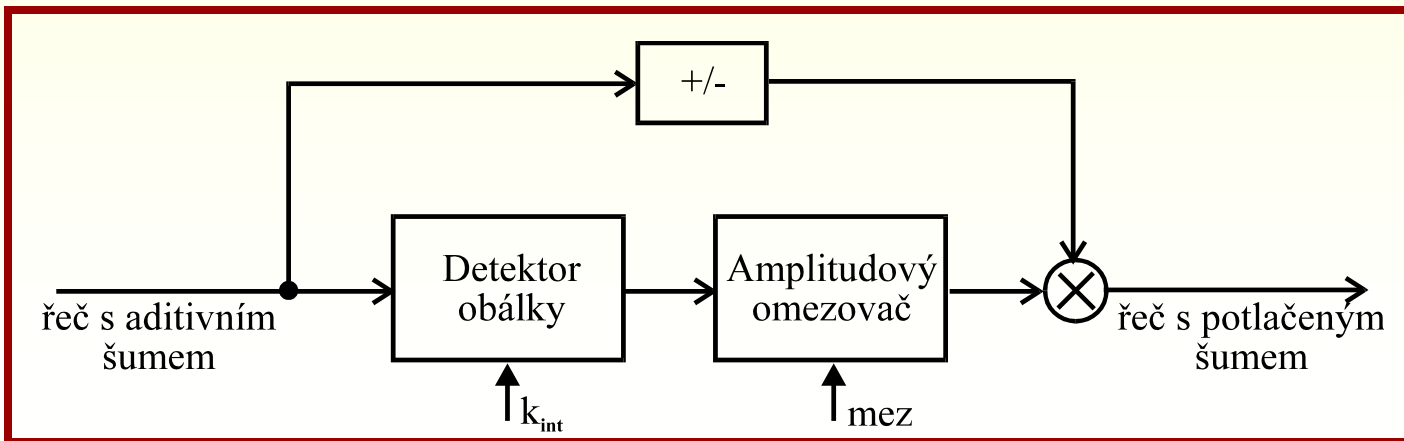
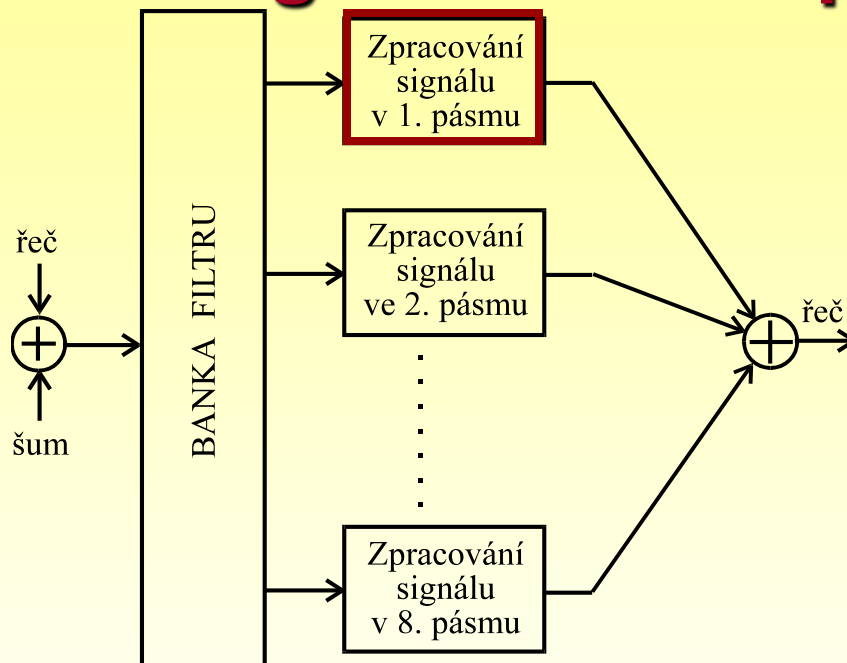
## banka filtrů

Dílci normované propusti

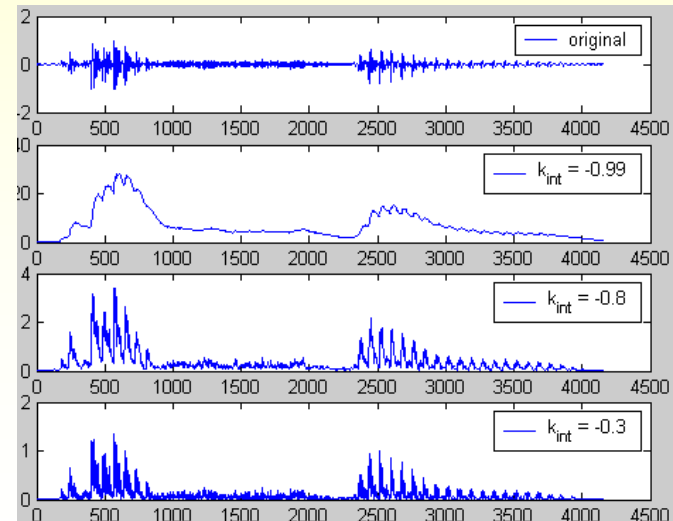
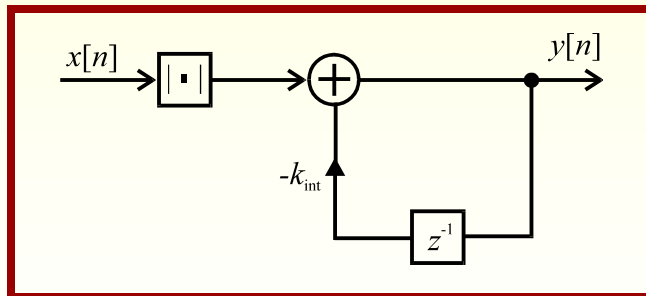
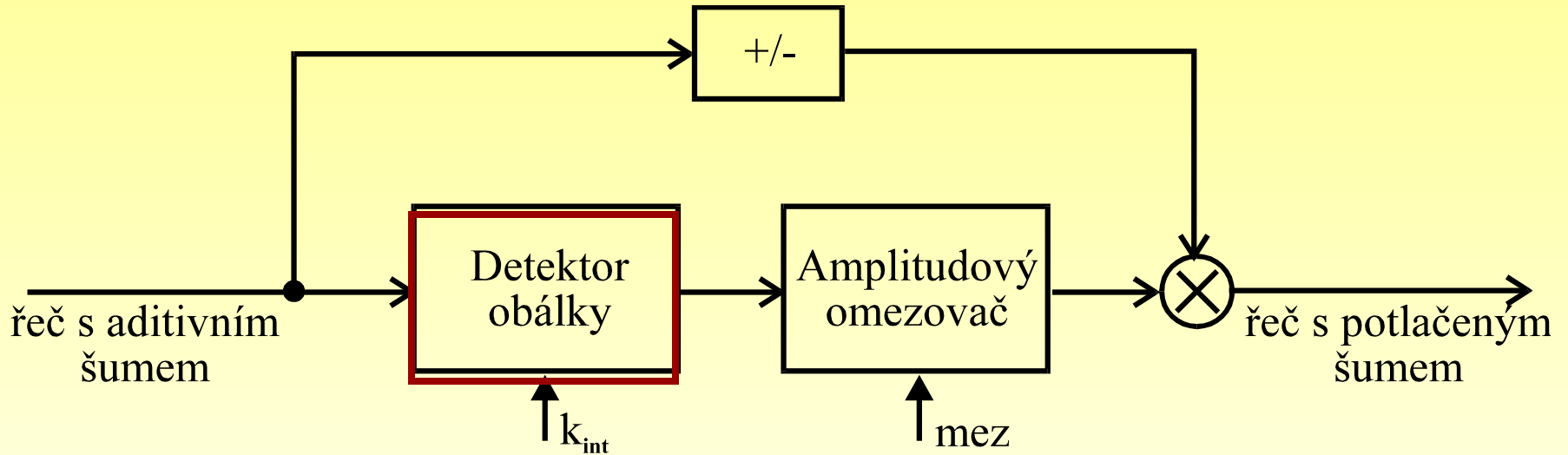


# Potlačení šumu v řeči bankou filtrů

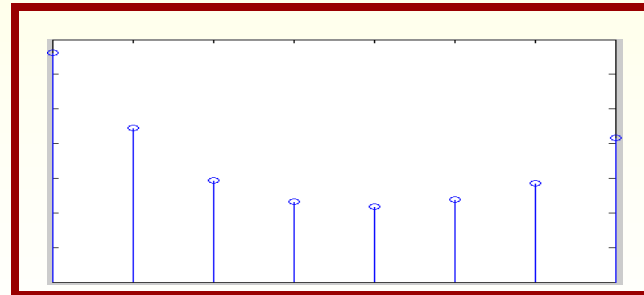
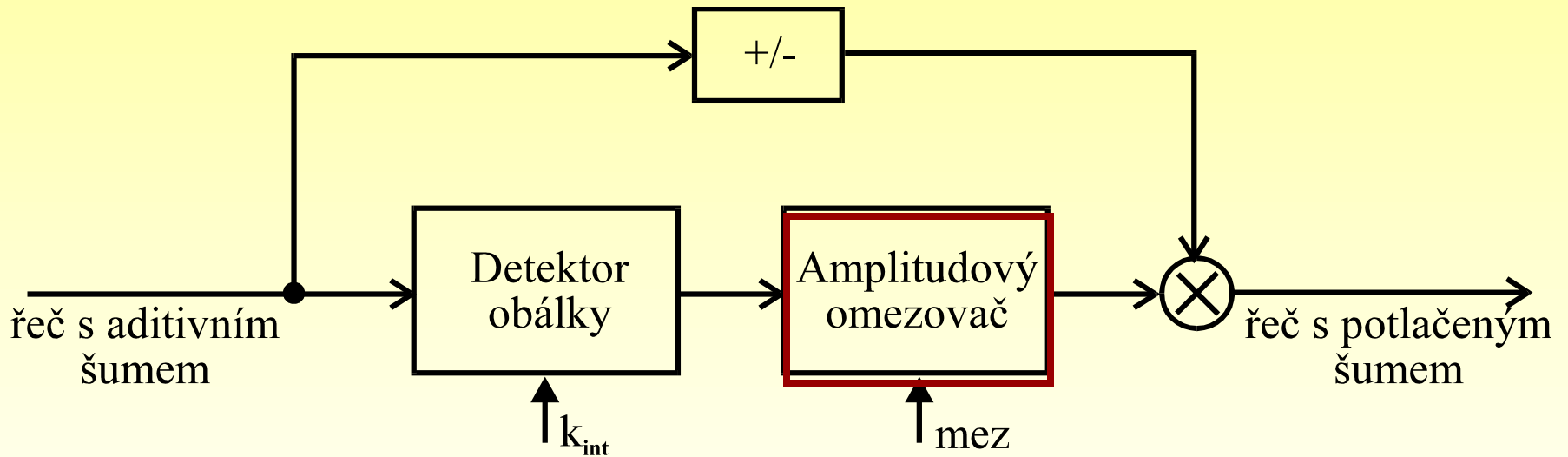
## zpracování signálů v i-tém pásmu



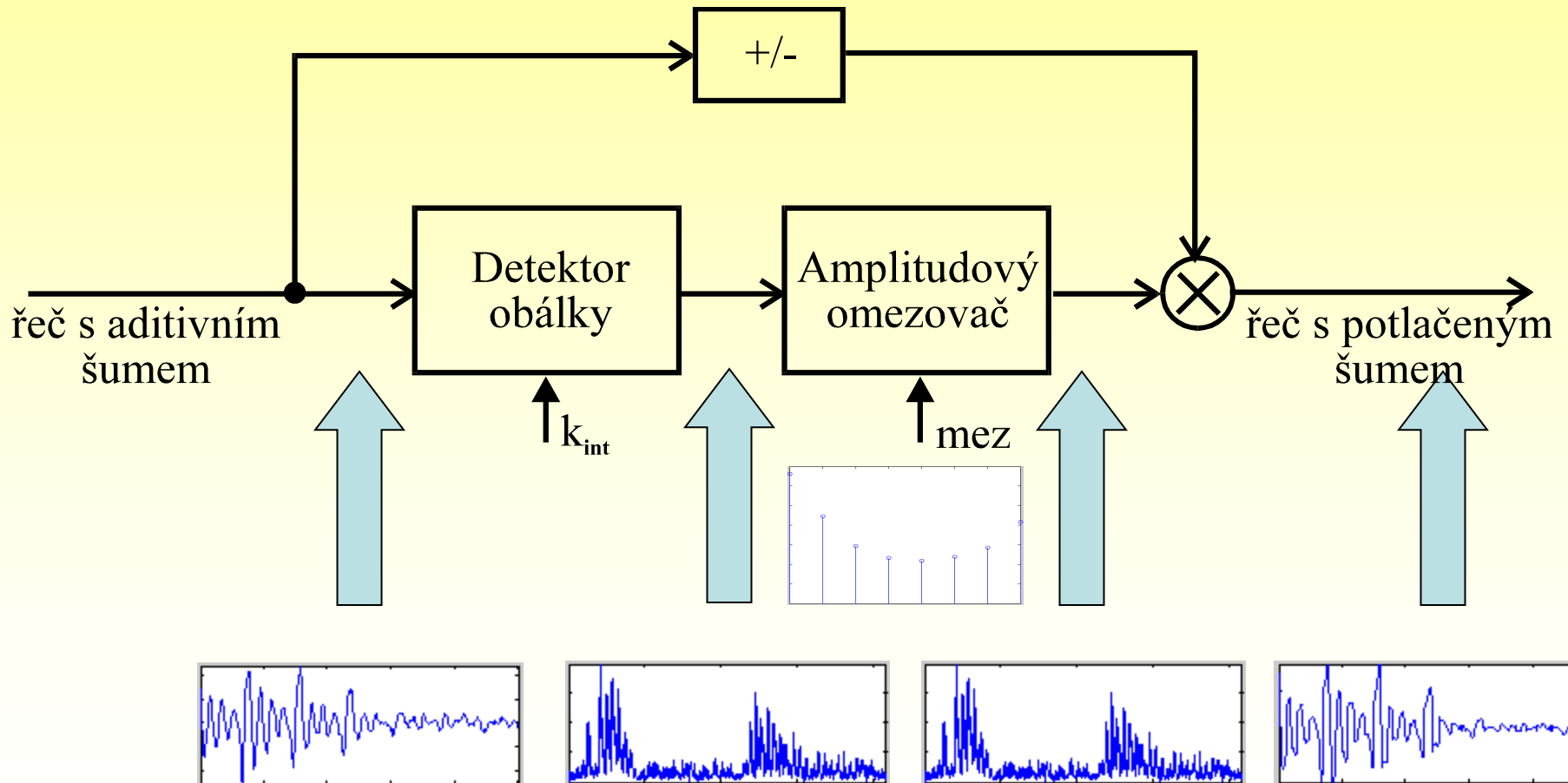
# Potlačení šumu v řeči bankou filtrů



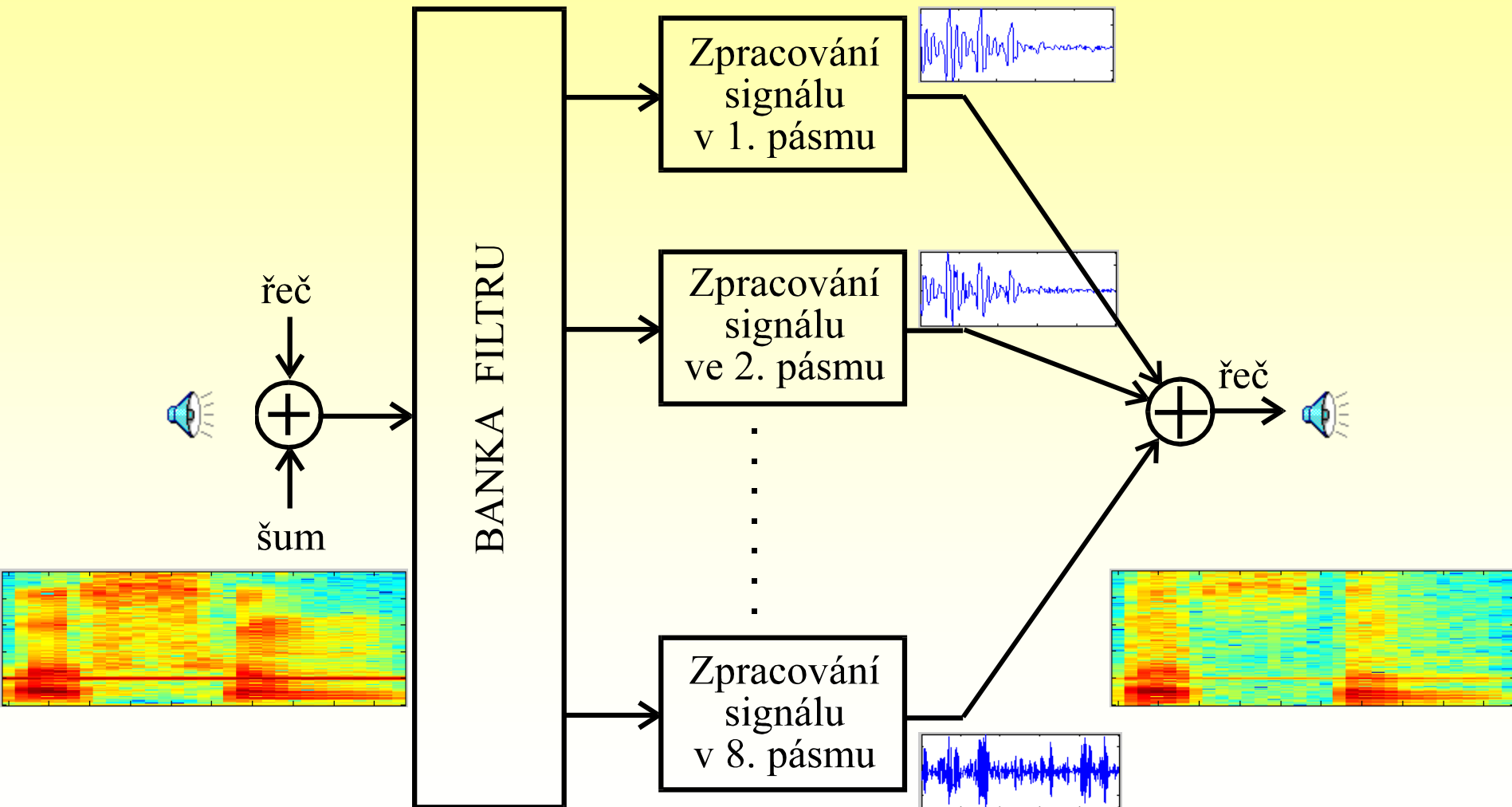
# Potlačení šumu v řeči bankou filtrů



# Potlačení šumu v řeči bankou filtrů



# Potlačení šumu v řeči bankou filtrů



# Potlačení šumu v řeči bankou filtrů III

## (generování rušení)

```
%%%%%%%%% GENEROVANI SUMU %%%%%%%%%%
k_noise= 0.1;

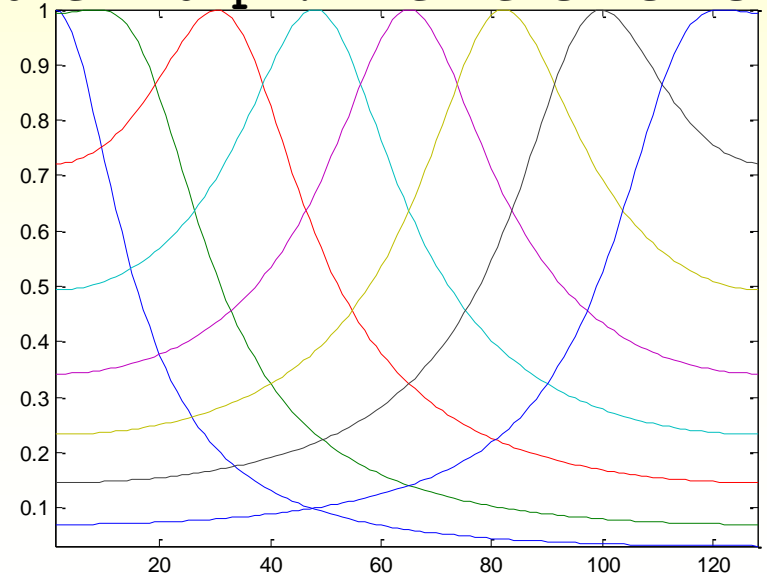
sig=input('1 = sinusove ruseni; 2 = nahodny sum ... ')

if sig==2,
    noise=k_noise*randn(length(xc),1);
end;
if sig==1,
    noise=sqrt(2)*k_noise*sin(2*pi*(0:length(xc)-1)*800/8000)';
end;
x=xc+noise;

'cinitel odstupu signalu k sumu'
snr=10*log10(sum(xc.^2)/sum(noise.^2))
```

# Potlačení šumu v řeči bankou filtrů IV (banka filtrů)

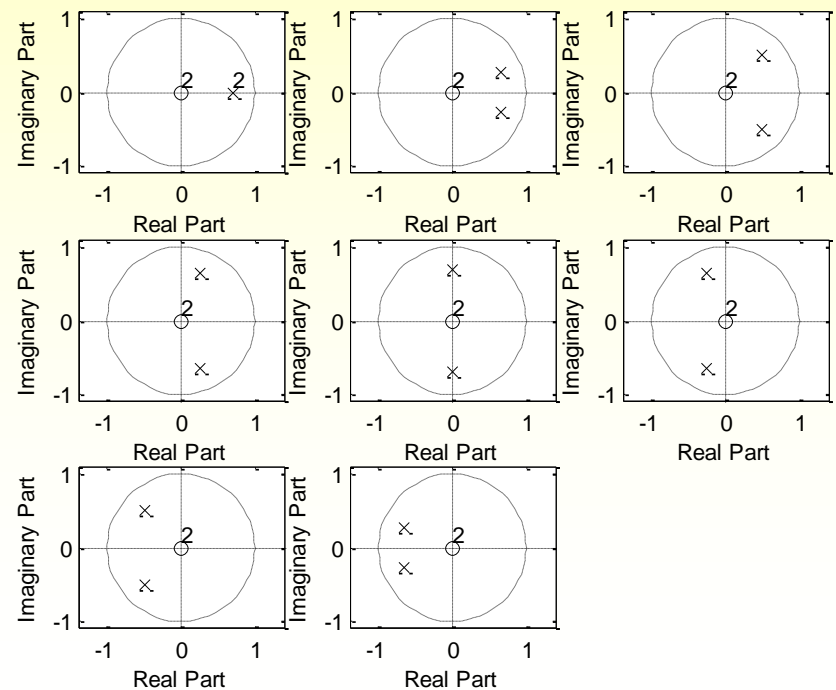
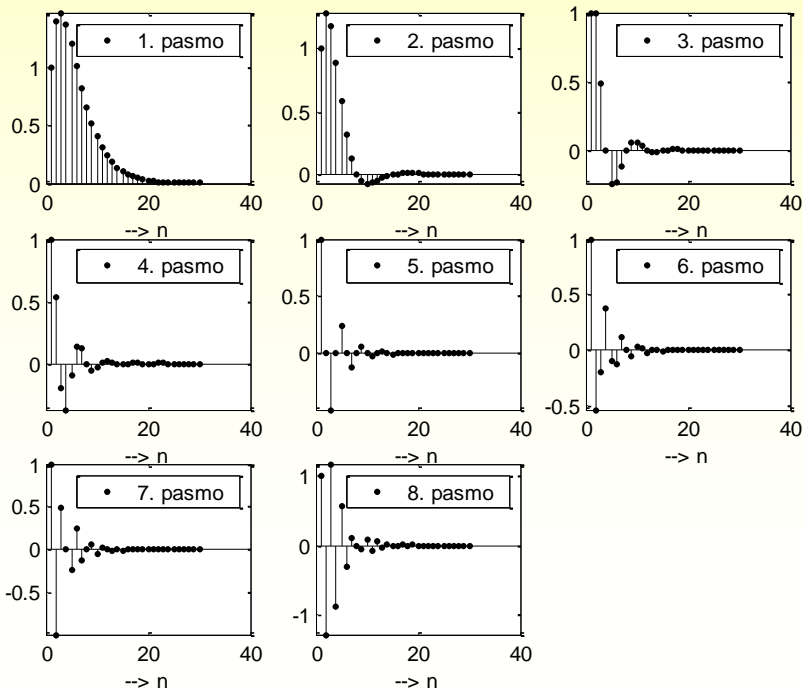
```
f_s=8; % vzorkovací frekvence [kHz]
P=8; % počet propustí
f_r=f_s/(2*P)*(0:P-1); % výpočet rezonančních frekvencí
R=0.7; % poloměr polu
b=-2*R*cos(2*pi*f_r/f_s); % výpočet koeficientu rezonátoru
for i=1:P % normování charakteristik
    Ha(:,i)=(freqz(1,[1 b(i) R.^2],128));
    Ham(i)=max(abs(Ha(:,i))); % výpočet normovacích koef.
    Han(:,i)=(freqz(1/Ham(i),[1 b(i) R.^2],128));
end;
plot(abs(Han)); axis tight % zobrazení amplitudových frekvencních ch.
title('Dílci normované propusti')
```



# Potlačení šumu v řeči bankou filtrů (banka filtrů)

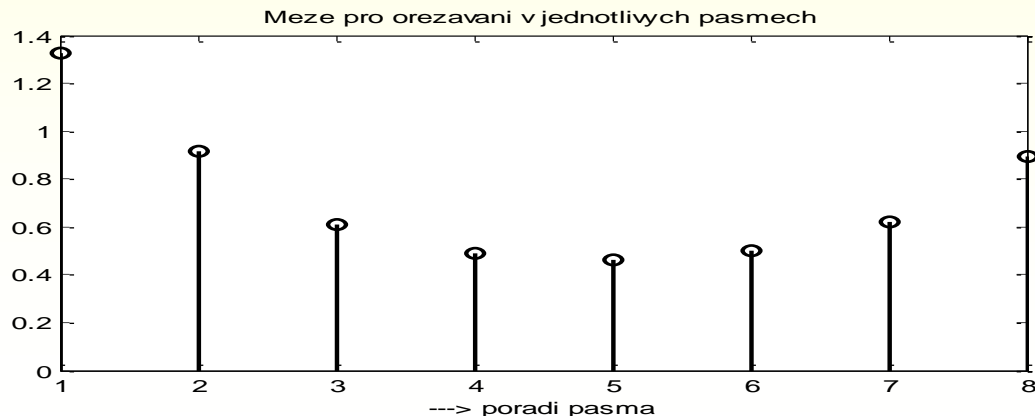
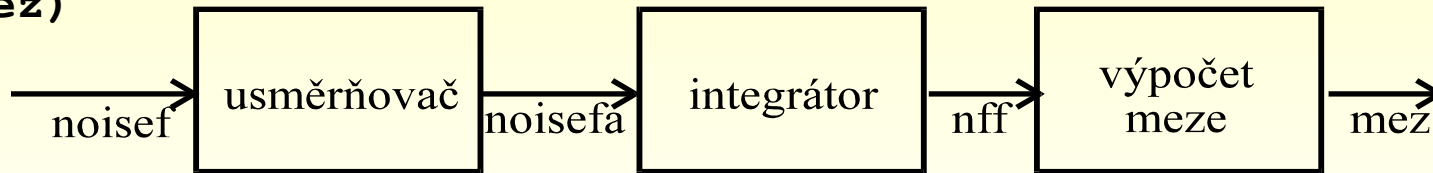
```
for k=1:P          % zobrazení impulsních charakteristik
    subplot(3,3,k), impz([1 0 0],[1 b(k) R.^2])
end;

% zobrazení z-roviny
for k=1:P
    subplot(3,3,k), zplane([1 0 0],[1 b(k) R.^2])
end;
```



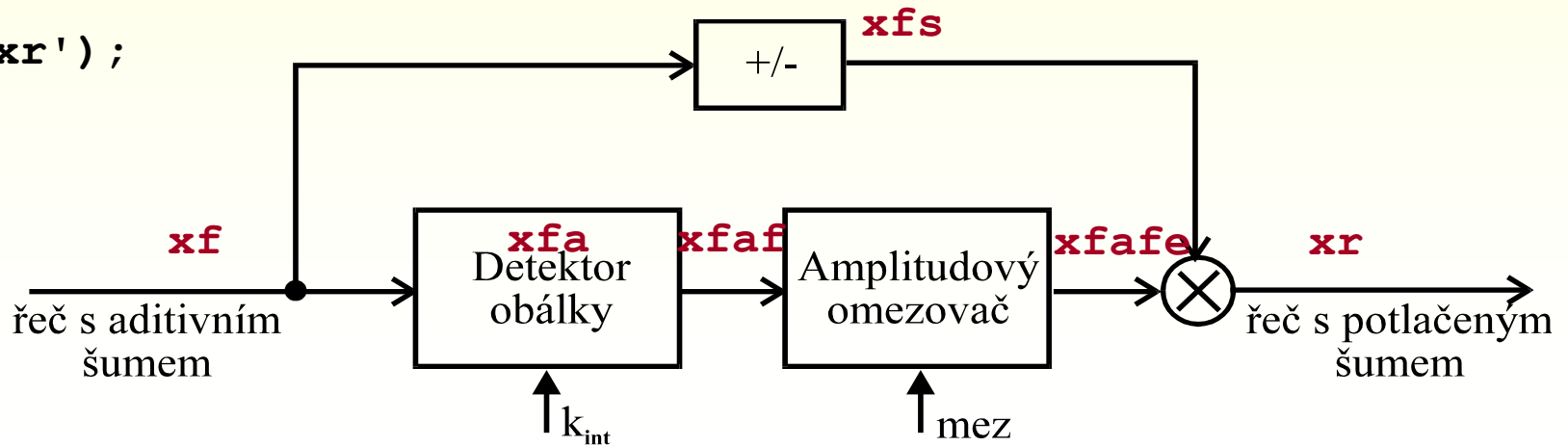
# Potlačení šumu v řeči bankou filtrů (stanovení mezí v pásmech)

```
%%%%%%%% FILTRACE A VYPOCET MEZE PRO OREZAVANI %%%%%%%%%  
k_noise= 0.1; k_int = 0.8;  
for i=1:P  
    noisef(:,i)=filter(1/Ham(i),[1 b(i) R.^2],noise(1:1000));  
                                                    % filtrace  
    noisefa(:,i)=abs(noisef(:,i));  
                                                    % usmerneni  
    nff(:,i)=filter(1,[1 -k_int],noisefa(:,i));  
                                                    % obalka  
    mez(i)=mean(abs(nff(:,i))); % vypocet mezi ve vseh pasmech  
end;  
stem(mez)
```

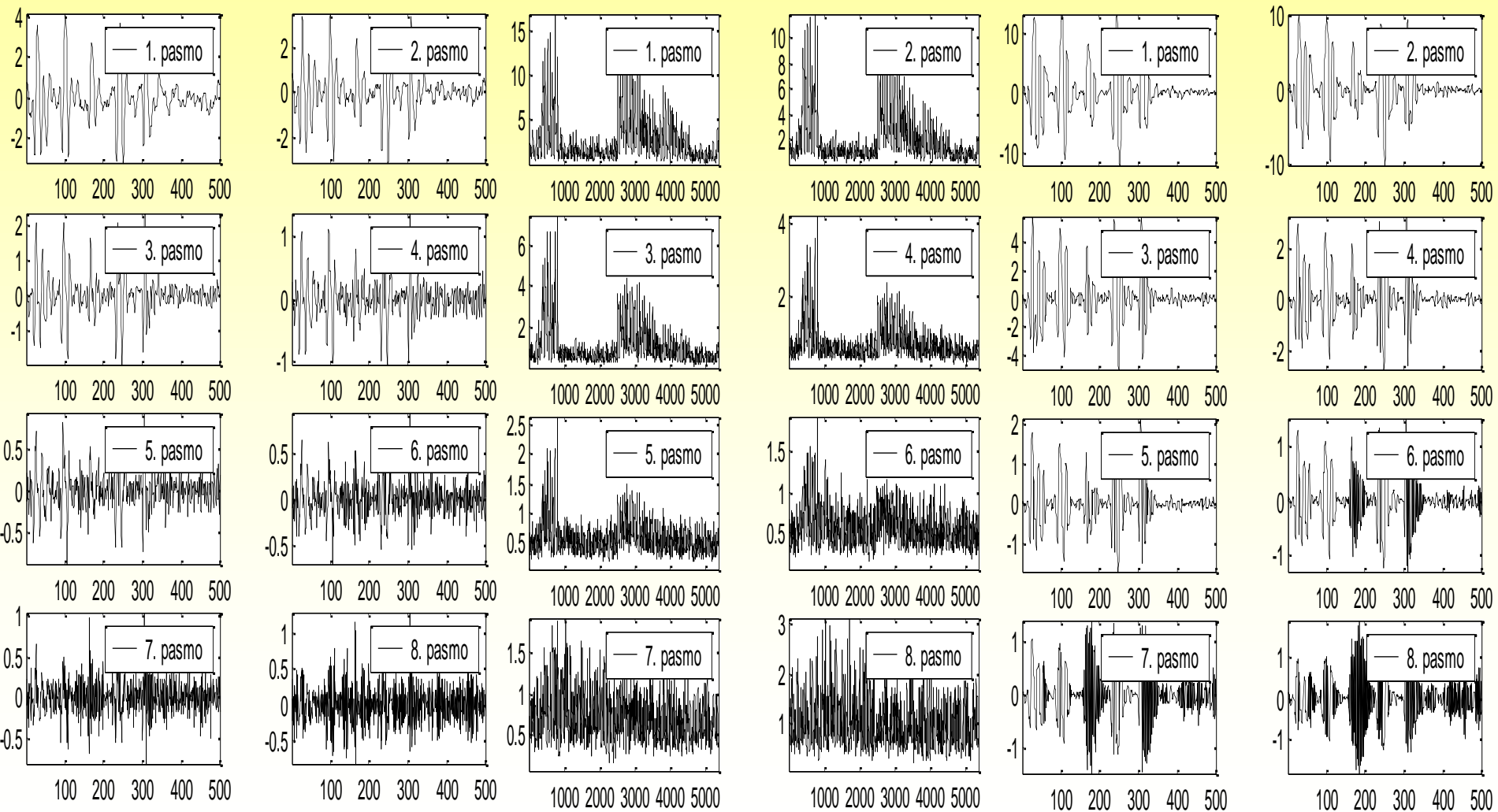


# Potlačení šumu v řeči bankou filtrů (filtrace a rozklad do pásem)

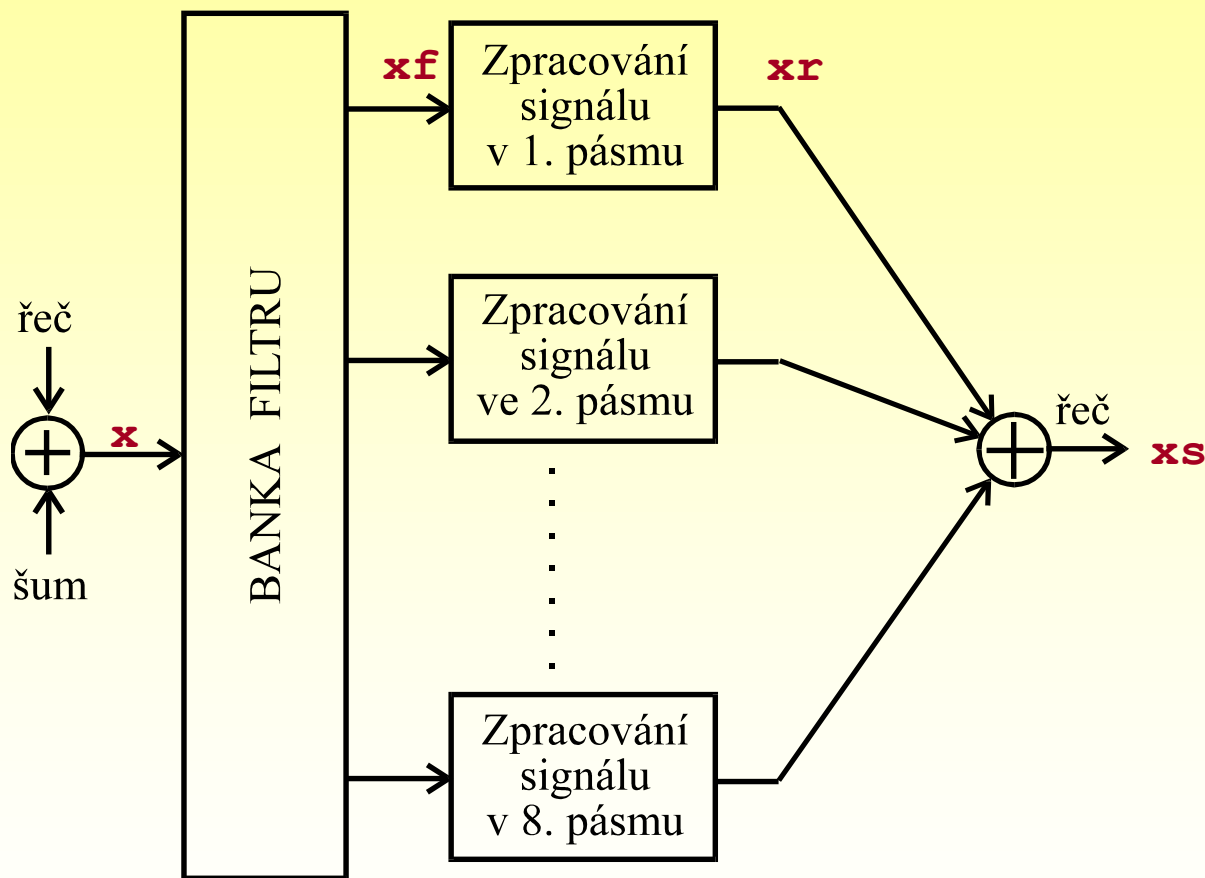
```
%%%%%%%%%% POTLACENI SUMU BANKOU FILTRU %%%%%%%%%%%  
%%% FILTRACE A ROZKLAD DO PASEM - VYPOCET OBALEK %%%%%%%%%%%  
for i=1:P  
    xf(:,i)=filter(1/Ham(i),[1 b(i) R.^2],x);    % filtrace  
    xfs(:,i)=sign(xf(:,i)+0.00001);              % znamenko  
    xfa(:,i)=abs(xf(:,i));                       % usmerneni  
    xfaf(:,i)=filter(1,[1 -k_int],xfa(:,i));    % obalka  
    subplot(4,2,i), plot(xfaf(:,i)); axis tight  
    xfafe(:,i)=abs(xfaf(:,i)-mez(i));           % oriznuti  
    xr(:,i)=xfafe(:,i).*xfs(:,i);  
    subplot(4,2,i), plot(xf(500:1000,i))  
    subplot(4,2,i), plot(xr(500:1000,i)); axis tight  
end;  
xs=sum(xr');
```



# Potlačení šumu v řeči bankou filtrů

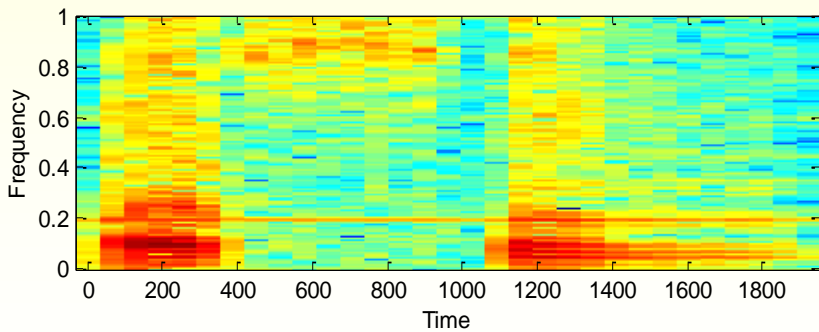
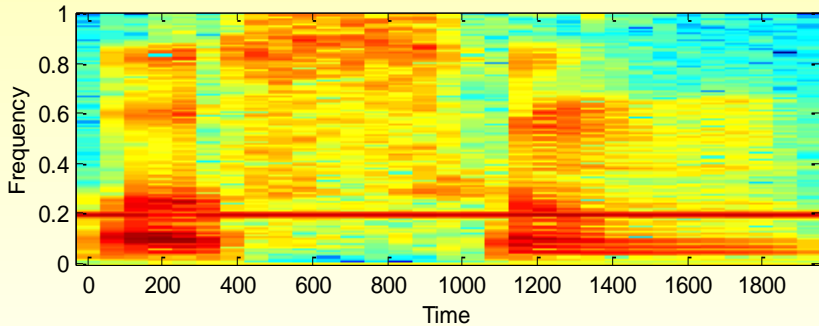


# Potlačení šumu v řeči bankou filtrů

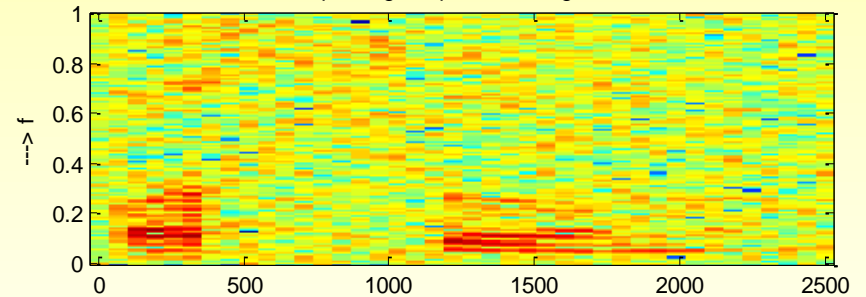


# Potlačení šumu v řeči bankou filtrů

```
for i=1:3 soundsc(x); pause(1), soundsc(xs); pause(1);end;  
subplot(211); specgram(x); subplot(212); specgram(xs);
```



Spektrogram původního signálu



Spektrogram signálu s potlačeným šumem

